KNOWLEDGE ORGANISER: Year 12 Statistics – Chapter 1 – Data Collection

A Data Set

- A **population** is the whole set of items that are of interest.
- A **census** observes or measures every member of a population

	Advantages	Disadvantages
Census	• It should give a completely accurate result	• Time consuming and expensive • Cannot be used when the testing process destroys the item • Hard to process large quantity of data
Sample	• Less time consuming and expensive than a census • Fewer people have to respond • Less data to process than in a census	• The data may not be as accurate • The sample may not be large enough to give information about small sub-groups of the population

Samples

A **sample** is a selection of observations taken from a subset of the population which is used to find out information about the population as a whole

Sampling

Individual units of a population are known as **sampling units.** Often sampling units of a population are individually named or numbered to form a list called a **sampling frame**

A simple random sample of size *n* is one where every sample of size *n* has an equal chance of being selected. To carry out a simple random sample, you need a sampling frame and each person or thing is allocated a unique number and a selection of these numbers is chosen at random

Advantages	Disadvantages
• Free of bias • Easy and cheap to implement for small populations and small samples • Each sampling unit has a known and equal chance of selection	• Not suitable when the population size or the sample size is large as it is potentially time consuming, disruptive and expensive. • A sampling frame is needed

In systematic sampling, the required elements are chosen at regular intervals from an ordered list

Advantages	Disadvantages
• Simple and quick to use • Suitable for large samples and large populations	• A sampling frame is needed • It can introduce bias if the sampling frame is not random

In stratified sampling, the population is divided into mutually exclusive strata, e.g. males and females, and a random sample is taken from each. The number sampled in a stratum equals

$$\frac{\text{number in stratum}}{\text{number in population}} \times \text{overall sample size}$$

Advantages	Disadvantages
• Sample accurately reflects the population structure • Guarantees proportional representation of groups within a population	• Population must be clearly classified into distinct strata • Selection within each stratum suffers from the same disadvantages as simple random sampling

In **quota sampling**, an interviewer or researcher selects a sample that reflects the characteristics of the whole population

Advantages	Disadvantages
• Allows a small sample to still be representative of the population • No sampling frame required • Quick, easy and inexpensive • Allows for easy comparison between different groups within a population	• Non-random sampling can introduce bias • Population must be divided into groups, which can be costly or inaccurate • Increasing scope of study increases number of groups, which adds time and expense • Non-responses are not recorded as such

Opportunity sampling consists of taking the sample from people who are available at the time of study is carried out and who fit the criteria you are looking for

Advantages	Disadvantages
• Easy to carry out • Inexpensive	• Unlikely to provide a representative sample • Highly dependent on individual researcher

Variables

- **Quantitative variables** or **quantitative data** are variables or data associated with numerical observations
- **Qualitative variables** or **qualitative data** are variables or data associated with non-numerical observations
- **Continuous variables** are variables that can take any value in a given range
- **Discrete variables** are variables that can take only specific values in a given range

Large amounts of data can be displayed in a **frequency table** or **as grouped data**. When data is presented in a grouped frequency table, the specific data values are not shown. The groups are commonly known as classes.

- Class boundaries tell you the maximum and minimum values that belong in each class
- The midpoint is the average of the class boundaries
- The class width is the difference between the upper and the lower class boundaries

Large Data Set

- If you need to do calculations on the large data set in your exam, the relevant extract from the data will be provided.
- You will need to be familiar with the types and ranges of data in the large data set, and with the characteristics of each location. You need to be able to recall trends from within the data set, or identify a location based on given data.

A **measure of location** is a single value, which describes a position in a data set. If the single value describes the centre of the data, it is called a **measure of central tendency.**

The **mode** or modal class is the value or class that occurs most often

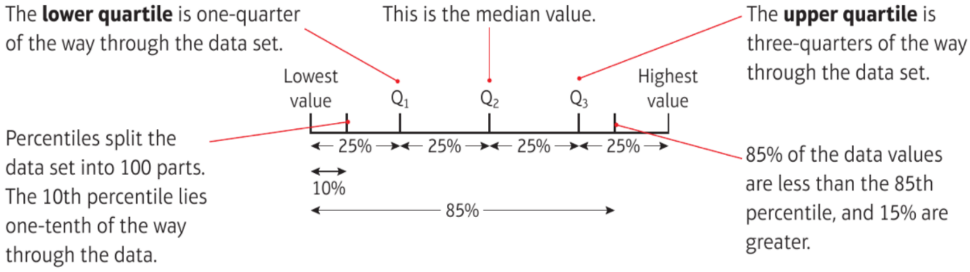
- The mean can be calculated using the formula $\bar{x} = \frac{\sum x}{n}$
- For data given in a frequency table, the mean can be calculated using the formula

$$\bar{x} = \frac{\sum xf}{\sum f}$$

The Median, Q_2

The median describes the middle of the data set. It splits the data set into two equal (50%) halves.

You can calculate other measures of location such as **quartiles** and **percentiles**.



- To find the **lower quartile, Q_1** for discrete data, divide n by 4. If this is a whole number, the lower quartile is halfway between this data point and the one above. If it is not a whole number, **round up** and pick this data point
- To find the **upper quartile, Q_3** for discrete data, find ¾n. If this is a whole number, the lower quartile is halfway between this data point and the one above. If it is not a whole number, **round up** and pick this data point

Coding is a way of simplifying statistical calculations. Each data value is coded to make a new set if data values which are easier to work with.

When data is coded, different statistics change in different ways. If data is coded using the formula

$$y = \frac{x-a}{b}$$

where a and b are constants you have to choose or are

- The **mean** of the coded data is given by

$$\bar{y} = \frac{\bar{x} - a}{b}$$

- **The standard deviation** of the of the coded data is given by

$$\sigma_y = \frac{\sigma_x}{b}$$

Where σ_x is the standard deviation of the original data

Measures of Spread

A measure of spread is a measure of how spread out the data is. The measures of spread are

- The **range** is the **difference between the largest and smallest values** in the data set. It takes into account **all of the data** and can be affected by extreme values
- **The interquartile range (IQR)** is the difference between the upper quartile and the lower quartile, $Q_3 - Q_1$. It is not affected by extreme values but only considers the spread of the middle 50% of the data
- The **inter-percentile range** is the **difference between the values for two given percentiles**. The 10th to 90th inter-percentile range is often used since it is not affected by extreme values but still considers 80% of the data in its calculation

Variance the spread of a data set can be used to work out.

$$\textbf{Variance} = \frac{\sum(x-\bar{x})^2}{n} = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = \frac{S_{xx}}{n}$$

The **Standard Deviation,** σ, is the square root of the variance

$$\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} = \sqrt{\frac{S_{xx}}{n}}$$

- **You can use these versions of the formulae for variance and standard deviation for grouped data that is presented in a frequency table:**
  - $\sigma^2 = \frac{\sum f(x-\bar{x})^2}{\sum f} = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2$
  - $\sigma = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$

  where f is the frequency for each group and $\sum f$ is the total frequency.

KNOWLEDGE ORGANISER: Year 12 Statistics – Chapter 3 – Representations of Data

Outliers & Anomalies

A common definition of an **Outlier** is any value that is:

- Either greater than $Q_3 + k(Q_3 - Q_1)$
- Or less than $Q_1 - k(Q_3 - Q_1)$

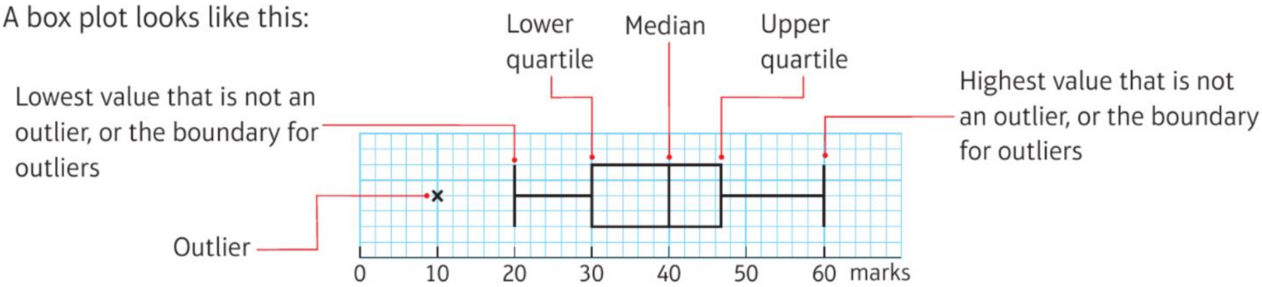
- There are occasions when an outlier should be removed from the data since it is clearly an error and it would be misleading to keep. There data values are known as **anomalies**

- The process of **removing anomalies** from a data set is known as **cleaning the data**

Box Plots

A box plot can be drawn to represent important features of the data. It shows the quartiles, maximum and minimum values and any outliers.

A box plot looks like this:



Two sets of data can be compared using box plots.

Cumulative Frequency

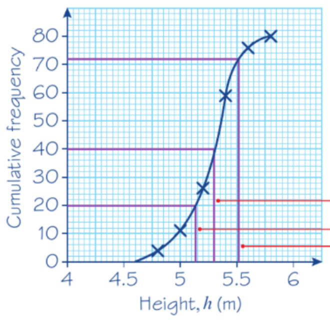
Data in a grouped frequency table does not able you to find the exact values of the median and quartiles. A **cumulative frequency diagram** is used to help find estimates for the median, quartiles and percentiles

Height, h (m)	Frequency	Cumulative frequency
$4.6 \leqslant h < 4.8$	4	4
$4.8 \leqslant h < 5.0$	7	11
$5.0 \leqslant h < 5.2$	15	26
$5.2 \leqslant h < 5.4$	33	59
$5.4 \leqslant h < 5.6$	17	76
$5.6 \leqslant h < 5.8$	4	80

$4 + 7 = 11$

$11 + 15 = 26$

This represents the number of data values that are in the range $4.6 \leqslant h < 5.4$.



The lowest possible value for the height is 4.6 m so plot (4.6, 0).

Plot each point using the upper class boundary for x and the cumulative frequency for y: coordinates (4.8, 4), (5.0, 11), (5.2, 26), (5.4, 59), (5.6, 76) and (5.8, 80). Join the points with a smooth curve.

For part **b**, draw a line across from 40 on the cumulative frequency axis and then down to the height axis.

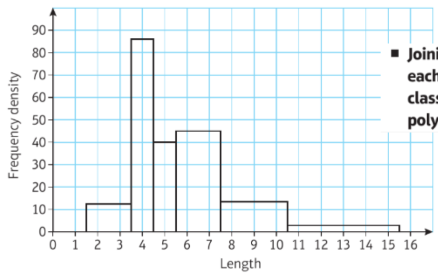
For part **c**, draw lines to estimate the lower quartile and the 90th percentile.

Histograms

Group continuous data can be represented in a **histogram**. It enables you to see a rough location, the general shape and how spread out the data is. The **area** of the bar is proportional to the frequency in each class so a histogram is used to represent grouped data with unequal class intervals.

- **On a histogram, to calculate the height of each bar (the frequency density) use the formula area of bar = k × frequency. $k = 1$ is the easiest value to use when drawing a histogram. If $k = 1$, then**

$$\text{frequency density} = \frac{\text{frequency}}{\text{class width}}$$



- **Joining the middle of the top of each bar in a histogram with equal class widths forms a frequency polygon.**

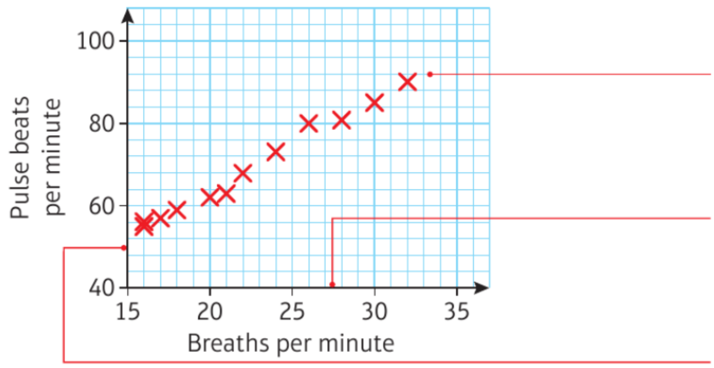
Comparing data sets you can comment on:

- A measure of location
- A measure of spread

You can compare data using the mean and standard deviation or using the median and the IQR. If the data set contains extreme values, then the median and IQR are more appropriate statistics to use.

- **Bivariate data is data which has pairs of values for two variables.**

You can represent bivariate data on a **scatter diagram**. This scatter diagram shows the results from an experiment on how breath rate affects pulse rate:



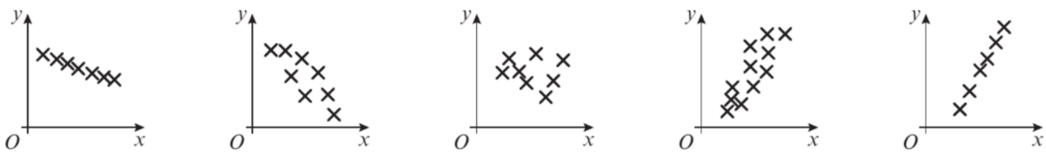
Each cross represents a data point. This subject had a breath rate of 32 breaths per minute and a pulse rate of 89 beats per minute.

The researcher could control this variable. It is called the **independent** or **explanatory variable**. It is usually plotted on the horizontal axis.

The researcher measured this variable. It is called the **dependent** or **response variable**. It is usually plotted on the vertical axis.

The two different variables in a set of bivariate data are often related.

- **Correlation describes the nature of the linear relationship between two variables.**



Strong negative correlation | Weak negative correlation | No (or zero) linear correlation | Weak positive correlation | Strong positive correlation

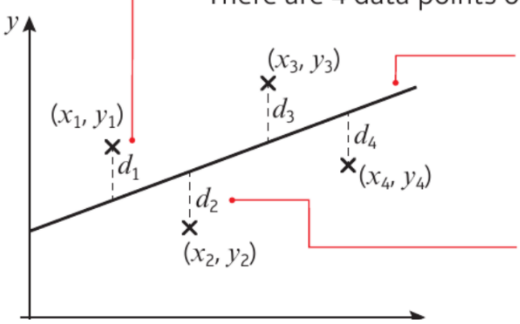
- For **negatively correlated** variables, when one variable **increases the other decreases**
- For **positively correlated** variables, when one variable **increases the other increases**

Two variables have a **causal relationship** if a change in one variable causes a change in the other. Just because two variables show correlation it does not mean that, they have a causal relationship.

- When two variables are correlated, you need to consider the context of the question and use your common sense to determine whether they have a causal relationship.

Least squares regression line usually known as the **regression line** is a type of line of best fit. It is a straight line, that minimises the sum of the squares of the distances of each data point from the line.

There are 4 data points on this scatter diagram.



The regression line of y on x is the straight line that minimises the value of $d_1^2 + d_2^2 + d_3^2 + d_4^2$. In general, if each data point is a distance d_i from the line, the regression line minimises the value of Σd_i^2.

The point (x_2, y_2) is a vertical distance d_2 from the line.

- **The regression line of y on x is written in the form $y = a + bx$.**

You can use a calculator to find the values of the coefficients a and b for a given set of bivariate data. You will not be required to do this in your exam.

- **The coefficient b tells you the change in y for each unit change in x.**
  - **If the data is positively correlated, b will be positive.**
  - **If the data is negatively correlated, b will be negative.**

<u>**Note:**</u>

- You should only use the regression line to make predictions for values of the dependent variable that are within the range of the data given
- The order of the variables is important. The regression line of y on x will be different from the regression line of x on y

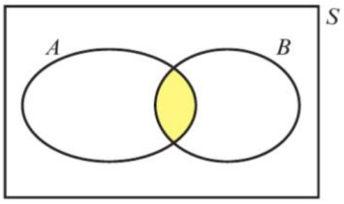
Calculating Probabilities

- If you want to predict the chance of something happening, you use probability
- An **experiment** is a repeatable process that gives rise to a number of **outcomes**
- An **event** is a collection of one or more outcomes
- A **sample space** is the set of all possible outcomes
- Where outcomes are **equally likely** the probability of an event is the number of outcomes in the event divided by the total number of possible outcomes.
- All events have a probability between 0 (impossible) and 1 (certain).
- Probabilities are usually written as fractions or decimals

Use a **Venn diagram** to represent events graphically. Place the frequencies or probabilities in the regions of the Venn diagram.

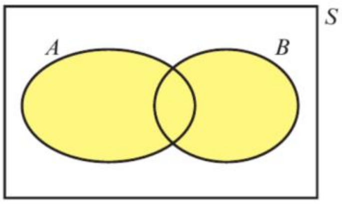
For events A and B in a sample space S:

1 The event A **and** B



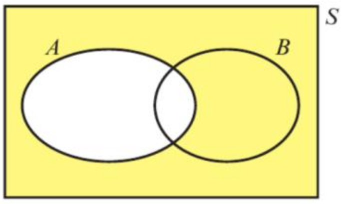
This event is also called the **intersection** of A and B. It represents the event that both A and B occur.

2 The event A **or** B



This event is also called the **union** of A and B. It represents the event that either A or B, or both, occur.

3 The event **not** A



This event is also called the **complement** of A. It represents the event that A does not occur.

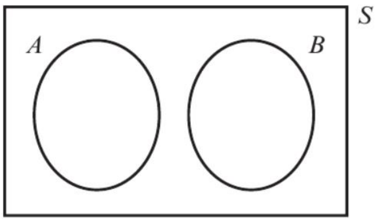
P(not A) = 1 – P(A)

You can write numbers of outcomes (frequencies) or the probability of the events in a Venn diagram to help solve problems.

When events have no outcomes in common they are called **mutually exclusive**.

In a Venn diagram, the closed curves do not overlap and you can use a simple addition rule to work out combined probabilities:

- **For mutually exclusive events, P(A or B) = P(A) + P(B).**



When one event has no effect on another, they are **independent**. Therefore if A and B are independent, the probability of A happening is the same whether or not B happens.

- **For independent events, P(A and B) = P(A) × P(B).**

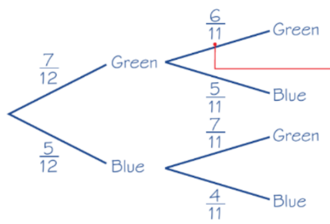
You can use this **multiplication rule** to determine whether events are independent.

A **tree diagram** is used to show the outcomes of two (or more) events happening in succession.

Example

A bag contains seven green beads and five blue beads. A bead is taken from the bag at random and not replaces. A second bead is then taken from the bag. Find the probability that
a) both beads are green
b) the beads are different colours.

a



Draw a tree diagram to show the events.

There are now only 6 green beads and 11 beads in total.

P(green and green) $= \frac{7}{12} \times \frac{6}{11} = \frac{7}{22}$

Multiply along the branch of the tree diagram.

b P(different colours)

= P(green then blue) + P(blue then green)

$= \frac{7}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{7}{11} = \frac{35}{66}$

Multiply along each branch and add the two probabilities.

KNOWLEDGE ORGANISER: Year 12 Statistics – Chapter 6 – Statistical Distributions

A **random variable** is a variable whose value depends on the outcome of a random event.

- Then range of values that a random variable can take is called its **sample space**
- A **variable** can take any range of specific values.
- The variable is **discrete** if it can only take *certain* numerical values.

The variable is **random** if the outcome is not known until the experiment is carried out.

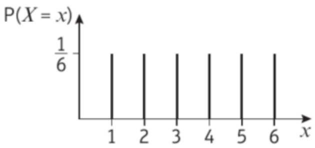
- **A probability distribution fully describes the probability of any outcome in the sample space.**

The probability distribution for a discrete random variable can be described in a number of different ways. For example, take the random variable X = 'score when a fair dice is rolled'. It can be described:

- as a **probability mass function**: $P(X = x) = \frac{1}{6}$, $x = 1, 2, 3, 4, 5, 6$
- using a table:

x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

- using a diagram:



- **The sum of the probabilities of all outcomes of an event add up to 1. For a random variable X, you can write $\sum P(X = x) = 1$ for all x.**

All of these representations show the probability that the random variable takes any given in its sample space.
When all of the probabilities are the same, as in this example, the distribution is known as a **discrete uniform distribution**

The Binomial Distribution

When you are carrying out a number of trials in an experiment or survey, you can define a random variable X to represent the number of **successful trials**.

- **You can model X with a binomial distribution, $B(n, p)$, if:**
  - **there are a fixed number of trials, n**
  - **there are two possible outcomes (success and failure)**
  - **there is a fixed probability of success, p**
  - **the trials are independent of each other**
- **If a random variable X has the binomial distribution $B(n, p)$ then its probability mass function is given by**

$$P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r}$$

n is sometimes called the **index** and p is sometimes called the **parameter**.

You can use your calculator to work out binomial probabilities. You can either use the rule given above, together with the ^nC_r function, or use the binomial probability distribution function directly.

A **cumulative probability function** for a random variable X tells you the sum of all the individual probabilities up to and including the given value of x in the calculation $P(X \leq x)$.
For the binomial distribution $X \sim B(n, p)$ there are tables in the formula book giving $P(X \leq x)$ for various values of n and p.

An extract from the tables is shown below:

$p =$	0.05	0.10	0.15	0.20	0.25	0.30
$n = 5, x = 0$	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681
1	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282
2	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369
3	1.0000	0.9995	0.9978	0.9933	0.9844	0.9692
4	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976
$n = 6, x = 0$	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176

For the binomial distribution $X \sim B(5, 0.3)$, this tells you that $P(X \leq 2) = 0.8369$ to 4 d.p.

You can also use the binomial cumulative probability function on your calculator to find $P(X \leq x)$ for any values of x, n and p.

A **hypothesis** is a statement made about the value of a **population parameter.** You can test a hypothesis about a population by carrying out an experiment or taking a sample from the population.
The result of the experiment or the statistic that is calculated from the sample is called the **test statistic.**
In order to carry out the test, you need to form two hypotheses:

- **The null hypothesis H_0, is the hypothesis that you assume to be correct.**
- **The alternative hypothesis H_1, tells you about the parameter if your assumption is shown to be wrong.**

- **Hypothesis tests with alternative hypotheses in the form $H_1: p < \cdots$ and $H_1: p > \cdots$ are called one-tailed tests.**
- **Hypothesis tests with an alternative hypothesis in the form $H_1: p \neq \cdots$ are called two-tailed tests.**

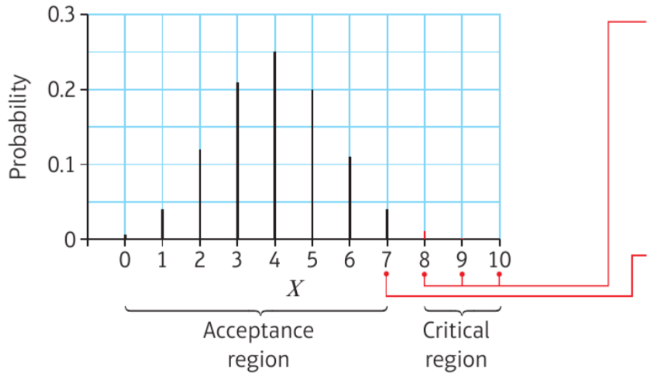
To carry out hypothesis test you **assume the null hypothesis is true,** and then consider how likely observed value of the test statistic was to occur. If this likelihood is less than a given threshold, called the **significance level** of the rest of the test, then you reject the null hypothesis. Typically, the significance level for a hypothesis test will be 10%, 5% or 1% but you will be told which level to use in the question.

Finding Critical Values

When you carry out a hypothesis test, you need to be able to calculate the probability of the test statistic taking particular values given that the null hypothesis is true.

- **A critical region is a region of the probability distribution which, if the test statistic falls within it, would cause you to reject the null hypothesis.**

A test statistic is modelled as $B(10, p)$ and a hypothesis test at the 5% significance level uses $H_0: p = 0.4$, $H_1: p > 0.4$. Assuming H_0 to be true, X has the following distribution: $X \sim B(10, 0.4)$:



$P(X = 10) = 0.0001$, $P(X = 9) = 0.0016$ and $P(X = 8) = 0.0106$. Hence $P(X \geqslant 8) = 0.0123$. This is less than the significance level of 5%.
A test statistic of 10, 9 or 8 would lead to the null hypothesis being rejected.

$P(X = 7) = 0.0425$. Adding this probability to $P(X \geqslant 8)$ takes the probability over 0.05 so a test statistic of 7 or less would lead to the null hypothesis being accepted.

- **The critical value is the first to fall inside of the critical region**

The critical value and hence the critical region can be determined from binomial distribution tables, or by finding cumulative binomial probabilities using your calculator.

- **The actual significance level of a hypothesis test is the probability of incorrectly rejecting the null hypothesis**
- **For a two-tailed test, there are two critical regions: one at each end of the distribution.**

One-tailed tests

If you have to carry out a one-tailed hypothesis test you need to:

- Formulate a model for the test statistic
- Identify suitable null and alternative hypotheses
- Calculate the probability of the test statistics taking the observed value (or higher/lower), assuming the null hypothesis is true
- Compare this to the significance level
- Write a conclusion in the context of the question

Alternatively, you can find the critical region and see whether the observed value of the test statistic lies inside it.

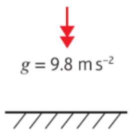
One-tailed tests

A one-tailed test is used when it is claimed that the probability has either gone up, or gone down. A two-tailed test is used when it is thought that the probability has changed in either direction.

- **For a two-tailed test, either double the *p*-value for your observation, or halve the significance level at the end you are testing.**

You need to know which tail of the distribution you are testing. If the test statistic is $X \sim B(n, p)$ then the **expected** outcome is np. If the observed value, x, is lower than this then consider $P(X \leq x)$. If the observed value is higher than the expected value, then consider $P(X \geq x)$.

KNOWLEDGE ORGANISER: Year 12 Mechanics – Chapter 8 – Modelling in Mechanics

Model	Modelling assumptions
Particle - dimensions of the object are negligible	• Mass of the object is concentrated at a single point • Rotational forces and air resistance can be ignored
Rod – all dimensions but one are negligible, like a pole of a beam	• Mass is concentrated along a line • No thickness • Rigid (does not bend or buckle)
Lamina - object with area but negligible thickness, like a sheet of paper	• Mass is distributed across a flat surface
Uniform body - mass is distributed evenly	• Mass of the object is concentrated at a single point at the geometrical centre of the body – the **centre of mass**
Light object - mass of the object is small compared to the other masses, like a string or a pulley	• Treat the object as having zero mass • Tension is the same at both ends of a light string
Inextensible string - a string that does bot stretch under load	• Acceleration is the same in objects connected by a taut inextensible string
Smoot surface	• Assume there is no friction between the surface and any object on it
Rough surface - if a surface is not smooth, it is rough	• Objects in contact with the surface experience a frictional force if they are moving or are acted on by a force
Wire - rigid thin length of metal	• Treated as one-dimensional
Smooth and light pulley - all pulleys you consider will be smooth and light	• Pulley has no mass • Tension is the same on either side of the pulley
Bead - particle with a hole in it for threading on a wire or string	• Moves freely along a wire or string • Tension is the same on either side of the bead
Peg - a support from which a body can be suspended or rested	• Dimensionless and fixed • Can be rough or smooth as specified in the question
Air resistance - resistance experienced as an object moves through the air	• Usually modelled as negligible
Gravity – force of attraction between all objects. Acceleration due to gravity is denoted by g. 	• Assume that all objects with mass are attracted towards the Earth • Earth's gravity is uniform and acts vertically downwards • G is constant and is taken as 9.8 ms^{-2}, unless otherwise stated in the question

These base SI units are most commonly used in mechanics:

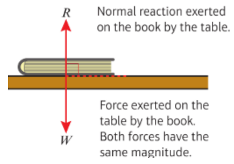
Quantity	Unit	Symbol
Mass	kilogram	kg
Length/displacement	metre	m
Time	seconds	s

These are derived units built form base units

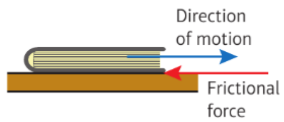
Quantity	Unit	Symbol
Speed/velocity	metres per second	m s^{-1}
Acceleration	metres per second per second	m s^{-2}
Weight/force	newton	N (= kg m s^{-2})

- **A vector is a quantity which has both magnitude and direction**
- **A scale quantity has magnitude only**

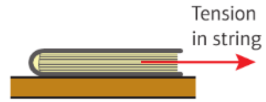
- The **weight** (or gravitational force) of an object acts vertically downwards
- The **normal reaction** is the force that acts perpendicular to a surface when an object is in contact with surface



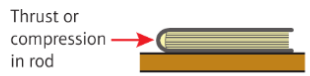
- The **friction** is a force which opposes the motion between two rough surfaces



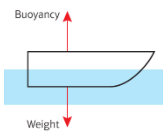
- If an object is being pulled along by a string, the force acting on the object is called the **tension** in the string



- If an object is being pushed along using a light rod, the force acting on the object is called the **thrust** or **compression** in the rod



- **Air resistance** opposes motion. In the case of a parachutist air resistance works vertically upwards to slow down the motion
- **Buoyance** is the upward force on a body that allows it to float or rise when submerged in a liquid

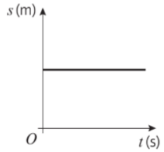


KNOWLEDGE ORGANISER: Year 12 Mechanics – Chapter 9 – Constant Acceleration

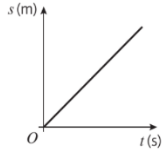
Displacement-time graphs

You can represent the motion of an object on a displacement–time graph. Displacement is always plotted on the vertical axis and time on the horizontal axis.

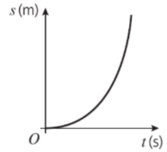
In these graphs s represents the displacement of an object from a given point in metres and t represents the time taken in seconds.



There is no change in the displacement over time and the object is stationary.



The displacement increases at a constant rate over time and the object is moving with constant velocity.

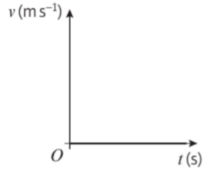


The displacement is increasing at a greater rate as time increases. The velocity is increasing and the object is accelerating.

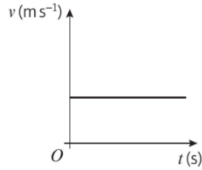
Velocity-time graphs

You can represent the motion of an object on a velocity–time graph. Velocity is always plotted on the vertical axis and time on the horizontal axis.

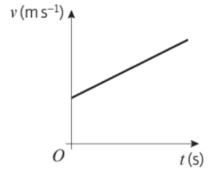
In these graphs v represents the velocity of an object in metres per second and t represents the time taken in seconds.



The velocity is zero and the object is stationary.



The velocity is unchanging and the object is moving with constant velocity.



The velocity is increasing at a constant rate and the object is moving with constant acceleration.

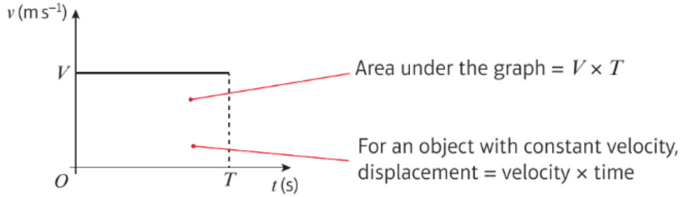
Velocity is the rate of change of displacement.

- **On a displacement-time graph the gradient represents the velocity.**
- **If the displacement-time graph is a straight line, then the velocity is constant**
- **Average Velocity=**$\dfrac{\textit{displacement from starting point}}{\textit{time taken}}$
- **Average speed =**$\dfrac{\textit{total distance travelled}}{\textit{time taken}}$

Acceleration is the rate of change of velocity

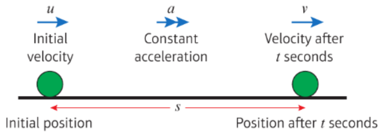
- **In a velocity-time graph the gradient represents the acceleration**
- **If the velocity-time graph is a straight line, then the acceleration is constant**

This velocity–time graph represents the motion of an object travelling in a straight line at constant velocity V m s^{-1} for time T seconds.



The area between a velocity-time graph and the horizontal axis represents the distance travelled.

- **For motion in a straight line with positive velocity, the area under the velocity-time graph up to a point *t* represents the displacement at time *t*.**



- s is the displacement.
- u is the initial velocity.
- v is the final velocity.
- a is the acceleration.
- t is the time.

You need to be able to use and to derive the five formulae for solving problems about particles moving in a straight line with constant acceleration.

- $v = u + at$
- $s = \left(\dfrac{u+v}{2}\right)t$
- $v^2 = u^2 + 2as$
- $s = ut + \dfrac{1}{2}at^2$
- $s = vt - \dfrac{1}{2}at^2$

Gravity

- **The force of gravity causes all objects to accelerate towards the earth. If you ignore the effects of air resistance, this acceleration is constant. It does depend on the weight of the object**
- **An object moving vertically in a straight line can be modelled as a particle with a constant downward acceleration of g = 9.8 ms^{-2}**

KNOWLEDGE ORGANISER: Year 12 Mechanics – Chapter 10 – Forces & Motion

Newton's first law of motion states that an object at rest will stay at rest and that an object moving with constant velocity will continue to move with constant velocity unless an unbalanced force acts on the object

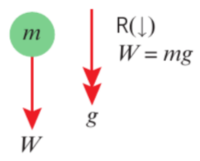
A **resultant** force acting on an object will cause to **accelerate in the same direction** as the resultant force.

Vectors

- You can find the resultant of two or more forces given as vectors by adding the vectors
- You can use *F=ma* to solve problems involving vector forces acting on particles
- In the vector form of F=ma, **F** and **a** are vectors. Acceleration as a 2D vector is written in the form ($p\mathbf{i} + q\mathbf{j}$) ms^{-2} or $\begin{pmatrix} p \\ q \end{pmatrix}$ ms^{-2}

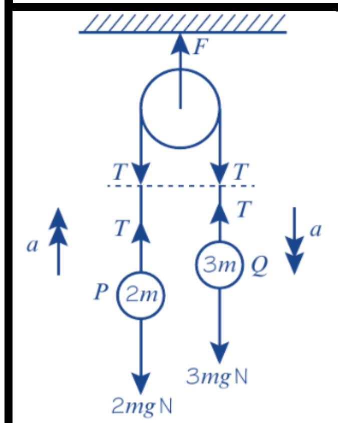
Newton's second law of motion states that the force needed to accelerate to a particle is equal to the product of the mass of the particle and the acceleration produced ***F = ma.***

Gravity is the force between any object and the Earth. The weight of the object is the force due to gravity acting on it, and it acts vertically downwards. A body falling freely experiences an acceleration of g = 9.8 ms^{-2}. Using the relationship *F = ma* gives ***W = mg***



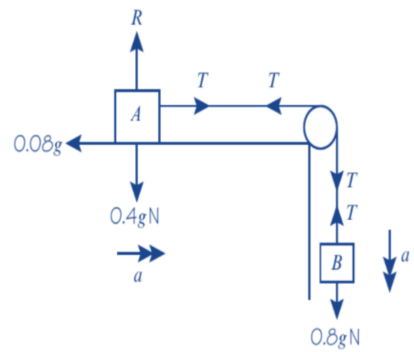
Newton's third law of motion states that for every action there is an equal and opposite reaction

Connected particles: you can solve problems involving connected particles by considering the particles separately or, if they are moving in the same straight line, as a single particle.
WARNING: particles need to remain in contact, or be connected by an inextensible rod or string to be considered as a single particle.



A smooth pulley means that the **tension in the string** will be the **same on both sides of the pulley.** The parts of the string each side of the pulley will be either horizontal or vertical

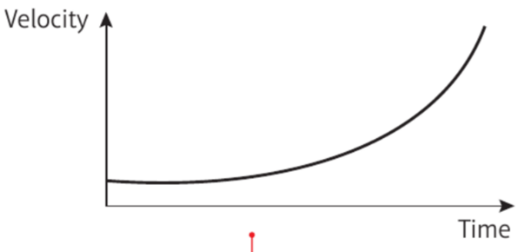
WARNING: you cannot treat a system involving a pulley as a single particle. This is because the particles are moving in different directions



KNOWLEDGE ORGANISER: Year 12 Mechanics – Chapter 11 – Variable Acceleration

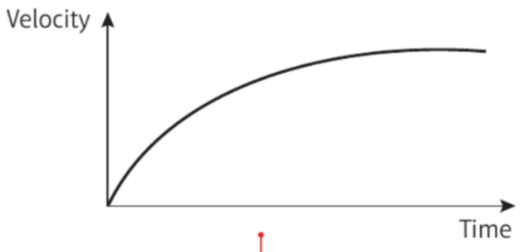
These velocity–time graphs represent the motion of a particle travelling in a straight line. They show examples of increasing and decreasing acceleration.

Increasing acceleration



The rate of increase of velocity is increasing with time and the gradient of the curve is increasing.

Decreasing acceleration



The rate of increase of velocity is decreasing with time and the gradient of the curve is decreasing.

Differentiate ↓ **Integrate** ↑

$$\text{displacement} = s = \int v \, dt$$

$$\frac{ds}{dt} = \text{velocity} = v = \int a \, dt$$

$$\frac{dv}{dt} = \frac{d^2s}{dt^2} = \text{acceleration} = a$$

Velocity is the rate of change of displacement.

- **If the displacement, s, is expressed as a function of t, then the velocity, v, can be expressed as $v = \frac{ds}{dt}$**
- **The gradient of a displacement-time graph represent velocity**

In the same way, acceleration is the rate of change of velocity.

- **If the velocity, v, is expressed as a function of time, t, then the acceleration, a, can be expressed as $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$**
- **The gradient of a velocity-time graph represents the acceleration**

Integration is the reverse of differentiation. You can integrate acceleration with respect to time to find velocity, and you can integrate velocity with respect to time to find displacement.