

PRIME NUMBERS

- 2 3 5 7 11 13 17 19 23 29 31 37 41 43...

The **FACTORS** of a number are all the numbers that **divide into it**.

There's a method that guarantees you'll find them all:

- 1) Start off with $1 \times$ the number itself, then try $2 \times$, then $3 \times$ and so on, listing the pairs in rows.
- 2) Try each one in turn. Cross out the row if it doesn't divide exactly.
- 3) Eventually, when you get a number **repeated**, **stop**.
- 4) The numbers in the rows you haven't crossed out make up the list of factors.

EXAMPLE:

Find all the factors of 24.

1×24
2×12
3×8
4×6
$5 \times$
6×4

So the factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24

Indices are the numbers written above a base number. E.g. For 8^2 – 8 is the base number and 2 is the index

The **index** shows how many times it has been multiplied by itself. E.g. $3^7 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

- 1) When **MULTIPLYING**, you **ADD THE POWERS**.
e.g. $3^6 \times 3^4 = 3^{6+4} = 3^{10}$, $a^2 \times a^7 = a^{2+7} = a^9$
- 2) When **DIVIDING**, you **SUBTRACT THE POWERS**.
e.g. $5^4 \div 5^2 = 5^{4-2} = 5^2$, $b^8 \div b^5 = b^{8-5} = b^3$

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

Long multiplication

24 x 16 becomes

2	4
24	
×	16
240	
144	
384	

Answer: 384

124 x 26 becomes

1	2	4
124		
×	26	
1240		
744		
3224		
1	1	

Answer: 3224

DIRECTED NUMBERS

Example 1: $-5 + -3$

Where the signs in the middle are different, the resulting sum becomes a subtraction so the above example would be $-5 - 3$ which equals -8 .

Example 2: $-6 - -3$

Where the signs in the middle are the same, the resulting sum becomes an addition so the above example would be $-6 + 3$ which equals -3 .

DECIMAL ADDITION

Example:
 $0.32 + 12.965 + 1.1$

Line up the decimal points	0.320	'Pad' with zeros
	12.965	
	+ 1.100	
	14.385	

Square numbers

- $1^2 = 1 \times 1 = 1$
- $2^2 = 2 \times 2 = 4$
- $3^2 = 3 \times 3 = 9$
- $4^2 = 4 \times 4 = 16$
- $5^2 = 5 \times 5 = 25$
- $6^2 = 6 \times 6 = 36$
- $7^2 = 7 \times 7 = 49$
- $8^2 = 8 \times 8 = 64$
- $9^2 = 9 \times 9 = 81$
- $10^2 = 10 \times 10 = 100$

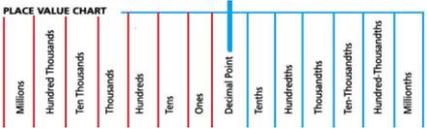
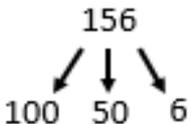
Cube numbers

- $1^3 = 1 \times 1 \times 1 = 1$
- $2^3 = 2 \times 2 \times 2 = 8$
- $3^3 = 3 \times 3 \times 3 = 27$
- $4^3 = 4 \times 4 \times 4 = 64$
- $5^3 = 5 \times 5 \times 5 = 125$
- $6^3 = 6 \times 6 \times 6 = 216$
- $7^3 = 7 \times 7 \times 7 = 343$
- $8^3 = 8 \times 8 \times 8 = 512$
- $9^3 = 9 \times 9 \times 9 = 729$
- $10^3 = 10 \times 10 \times 10 = 1000$

Here are the square roots of all the perfect squares from 1 to 100.

- $\sqrt{1} = 1$ since $1^2 = 1$
- $\sqrt{4} = 2$ since $2^2 = 4$
- $\sqrt{9} = 3$ since $3^2 = 9$
- $\sqrt{16} = 4$ since $4^2 = 16$
- $\sqrt{25} = 5$ since $5^2 = 25$
- $\sqrt{36} = 6$ since $6^2 = 36$
- $\sqrt{49} = 7$ since $7^2 = 49$

KNOWLEDGE ORGANISER: Year 10 Developing – UNITS 10D, 10E, 10F

1. Place Value	The value of a digit that relates to its position or place in a number.	53 000 000 The value of the 3 is 3 million because it is in the millions column
2. Digits	The symbols we use to write a number.	456 has 3 digits
3. Place Value Columns	The position that the digit can be put into that's determines the value of that digit.	<p>PLACE VALUE CHART</p> 
4. Partitioning	Splitting a number up.	
5. Comparing Numbers	We use these symbols: = Equal to < Less than > Greater than ≥ Greater than or equal to	$3 < 6$ 3 is less than 6 $6 > 3$

	\leq Less than or equal to	6 is greater than 3
Simplifying fractions	Cancel down by dividing both numerator and denominator by a common factor. Using the highest common factor means less steps are necessary	$\frac{20}{50} = \frac{2}{5}$ dividing by HCF of 10
3. Place Value	The value of a digit that relates to its position or place in a number.	In 12.34 the digits represent 1 ten, 2 ones, 3 tenths and 4 hundredths respectively.
4. Decimal Places	The number of digits after the decimal place.	5.34 has 2 decimal places 0.0915 has 4 decimal places
2. Inequality symbols	$x > 2$ means x is greater than 2 $x < 3$ means x is less than 3 $x \geq 1$ means x is greater than or equal to 1 $x \leq 6$ means x is less than or equal to 6	State the integers that satisfy $-2 < x \leq 4$. -1, 0, 1, 2, 3, 4

Equivalent Fractions	Fractions which represent the same value	$\frac{2}{5} = \frac{4}{10} = \frac{20}{50}$
9. Expand	To expand a bracket, multiply each term in the bracket by the expression outside the bracket.	$3(m + 7) = 3m + 21$
9. Brackets Index Laws	When raising a power to another power, multiply the powers together. $(a^m)^n = a^{mn}$	$(y^2)^5 = y^{10}$ $(6^3)^4 = 6^{12}$ $(5x^6)^3 = 125x^{18}$
10. Notable Powers	$p = p^1$ $p^0 = 1$	$99999^0 = 1$

KNOWLEDGE ORGANISER: Year 10 Developing – Units 10G-10I

10. Fraction of an Amount	Divide by the bottom, times by the top	Find $\frac{2}{5}$ of £60 $60 \div 5 = 12$ $12 \times 2 = 24$
9. Comparing Fractions	To compare fractions, they each need to be rewritten so that they have a common denominator . Ascending means smallest to biggest . Descending means biggest to smallest .	Put in to ascending order: $\frac{3}{4}, \frac{2}{3}, \frac{5}{6}, \frac{1}{2}$ Equivalent: $\frac{9}{12}, \frac{8}{12}, \frac{10}{12}, \frac{6}{12}$ Correct order: $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$

Changing between mixed numbers and improper fractions:

Example: Convert $\frac{11}{4}$ to a mixed fraction.

Divide:

$$\rightarrow 11 \div 4 = 2 \text{ with a remainder of } 3$$

Write down the 2 and then write down the remainder (3) above the denominator (4).

Answer:

$$2 \frac{3}{4}$$

Example: Convert $\frac{10}{3}$ to a mixed fraction.

$$\frac{10}{3} = 10 \div 3 = 3 \text{ R } 1$$

$$= 3 \frac{1}{3}$$

Answer:

$$3 \frac{1}{3}$$

Rounding to the nearest 10, 100, 1000:

134.9 rounded to tens is 130

as the next digit (4) is less than 5

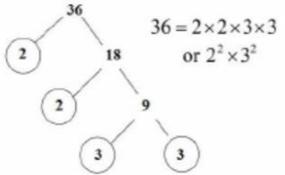
11. Adding or Subtracting Fractions	Find the LCM of the denominators to find a common denominator. Use equivalent fractions to change each fraction to the common denominator . Then just add or subtract the numerators and keep the denominator the same .	$\frac{2}{3} + \frac{4}{5}$ Multiples of 3: 3, 6, 9, 12, 15.. Multiples of 5: 5, 10, 15.. LCM of 3 and 5 = 15 $\frac{2}{3} = \frac{10}{15}$ $\frac{4}{5} = \frac{12}{15}$ $\frac{10}{15} + \frac{12}{15} = \frac{22}{15} = 1 \frac{7}{15}$
5. Simplifying Expressions	Collect 'like terms' . Be careful with negatives. x^2 and x are not like terms.	$2x + 3y + 4x - 5y + 3$ $= 6x - 2y + 3$ $3x + 4 - x^2 + 2x - 1 = 5x - x^2 + 3$
5. Substitution	Replace letters with numbers . Be careful of $5x^2$. You need to square first, then multiply by 5.	$a = 3, b = 2$ and $c = 5$. Find: 1. $2a = 2 \times 3 = 6$ 2. $3a - 2b = 3 \times 3 - 2 \times 2 = 5$ 3. $7b^2 - 5 = 7 \times 2^2 - 5 = 23$

Example: Find the least common multiple of 6 and 15:

The multiples of 6 are: **6, 12, 18, 24, 30, ...**
and the multiples of 15 are: **15, 30, ...**

There is a match at 30

So the least common multiple of 6 and 15 is **30**

7. Product of Prime Factors	Finding out which prime numbers multiply together to make the original number . Use a prime factor tree . Also known as 'prime factorisation'.	 $36 = 2 \times 2 \times 3 \times 3$ or $2^2 \times 3^2$
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12,690 rounded to thousands is 13,000

as the next digit (6) is 5 or more

To **Solve** an equation you find the value of the unknown by isolating it on one side by performing inverse operations. Remember that solutions can be integers, decimals, fractions and negative numbers.

$$\begin{aligned} 4x + 8 &= 24 \\ -8 & \quad -8 \\ \hline 4x + 0 &= 16 \\ \div 4 & \quad \div 4 \\ \hline x &= 4 \end{aligned}$$

Balancing method

$$\begin{aligned} 8a - 5 &= 11 \\ +5 & \quad +5 \\ \hline 8a &= 16 \\ +8 & \quad +8 \\ \hline a &= 2 \end{aligned}$$

Balancing method

$$\begin{aligned} 10 + 6y &= 34 \\ -10 & \quad -10 \\ \hline 6y &= 24 \\ \div 6 & \quad \div 6 \\ \hline y &= 4 \end{aligned}$$

Balancing method

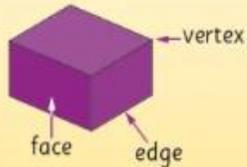
$$\begin{aligned} \frac{x}{12} - 5 &= 4 \\ +5 & \quad +5 \\ \hline \frac{x}{12} &= 9 \\ \times 12 & \quad \times 12 \\ \hline x &= 108 \end{aligned}$$

$$\begin{aligned} 5p - 7 &= 3p + 3 & -3p \\ 2p - 7 &= 3 & +7 \\ 2p &= 10 & \div 2 \\ p &= 5 \end{aligned}$$

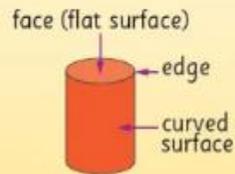
$$\begin{aligned} 2(10 - 2x) &= 4(2x + 2) \\ 20 - 4x &= 8x + 8 \\ 20 - 4x + 4x &= 8x + 8 + 4x \\ 20 &= 12x + 8 \\ 20 - 8 &= 12x + 8 - 8 \\ 12 &= 12x \\ \frac{12}{12} &= \frac{12x}{12} \\ 1 &= x \end{aligned}$$

- **Surface:** Can be flat (face) or curved.
- **Edge:** where two surfaces meet. An edge can be straight or curved.
- **Vertex:** where two or more edges meet at a point.

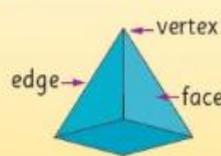
(a) Cuboid



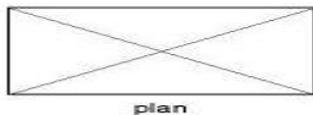
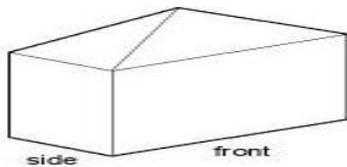
(b) Cylinder



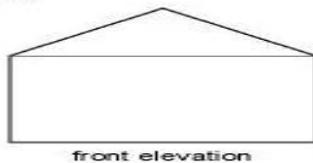
(c) Square pyramid



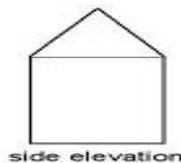
Maths Around Us video 3.4



View from above.



View from either Left or right side.



Percentages without a calculator

To work out 15% of £300...

Break up into 10% and 5%

$$\begin{aligned} 10\% \text{ of } \pounds 300 \\ \pounds 300 \div 10 &= \\ \pounds 30 \\ 10\% &= \pounds 30 \end{aligned}$$

$$\begin{aligned} 5\% \text{ of } \pounds 300 \\ \text{is half of } 10\% \\ 10\% &= \pounds 30 \\ 5\% &= \pounds 15 \end{aligned}$$

$$\begin{aligned} 15\% \text{ of } \pounds 300 &= 10\% + 5\% \\ &= \pounds 30 + \pounds 15 \\ &= \pounds 45 \end{aligned}$$

You can use a ratio table to organise your working out more clearly.

%	100%	10%	5%	15%
£	300	30	15	45

Note: Red arrows indicate +10 and +2 operations between columns.

Top Tip

There is no "correct" way to break up percentages. Use whatever amounts you find easiest.



Increase 540 by 36%

$$\begin{aligned} 10\% & \quad (\text{Divide by } 10) & \quad 54 \\ 5\% & \quad (\text{Divide by } 10\% \text{ by } 5) & \quad 27 \\ 1\% & \quad (\text{Divide by } 100 \text{ or divide } 10\% \text{ by } 10) & \quad 5.4 \\ 540 + (3 \times 54) + 27 + 5.4 &= 734.4 \end{aligned}$$

Decrease 470 by 16%

$$\begin{aligned} 10\% &= 47 \\ 5\% &= 23.5 \\ 1\% &= 4.7 \\ 470 - 47 - 23.5 - 4.7 &= 394.8 \end{aligned}$$