Regression Lines can be used to model a **linear relationship** between two variables. Sometimes, experimental data does not fit a linear model, but still shows a clear pattern. **Logarithms and coding** can be used to examine **trends in non-linear data.**

For data that can be modelled by a relationship of the form $y = ax^n$, you need to code the data using $Y = \log y$ and $X = \log x$ to obtain a linear relationship.

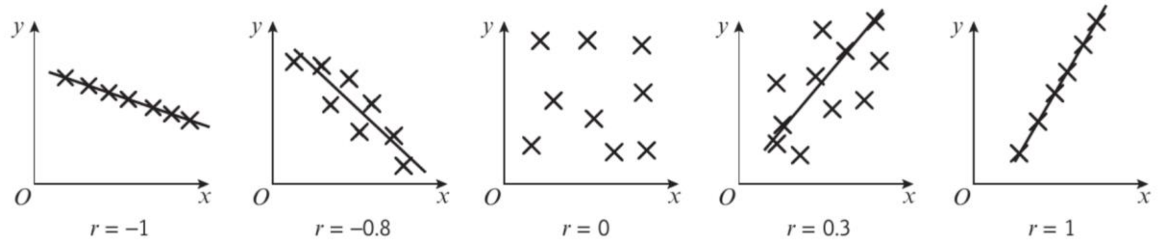
- **If $y = ax^n$ for constants a and n then $y = \log a + n\log x$**

For data that can be modelled by an **exponential** relationship of the form $y = ab^x$, you need to code the data using $Y = \log y$ and $X = x$ to obtain a linear relationship.

- **If $y = kb^x$ for constants k and b then $\log y = \log k + x\log b$**

The PRODUCT MOMENT CORRELATION COEFFICIENT (PMCC) describes the linear correlation between two variables. It can take values between -1 and 1

- If $r = 1$, there is perfect positive linear correlation
- If $r = -1$, there is perfect negative linear correlation
- The closer r is to -1 or 1, the stronger the negative or positive correlation, respectively
- If $r = 0$ (or is close to 0) there is no linear correlation. In this case, there might still be a non-linear relationship between the variables.



You need to know how to calculate the product moment correlation coefficient for bivariate data using your calculator.

HYPOTHESIS TESTING FOR ZERO CORRELATION

You can use a hypothesis test to determine whether the product moment correlation coefficient, r, for a particular sample indicates that there is likely to be a linear relationship within the whole population.

ρ denotes the product moment correlation coefficient for a **whole population.**

If you want to test for whether or not the population PMCC, ρ, is

- either greater that zero or less than zero you can use a **one-tailed test.**

For a one-tailed test either

- $H_0: \rho = 0, H_1: \rho > 0$ **or**
- $H_0: \rho = 0, H_1: \rho < 0$

- is not equal to zero you need to use a **two-tailed test:**

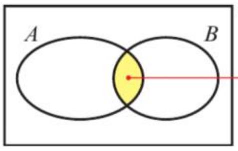
For a two-tailed test use

- $H_0: \rho = 0, H_1: \rho \neq 0$

Set notation can be used to desc … n help abbreviate probability statements

[Grab your reader's attention with a great quote from the document or use this space to emphasize a key point. To place this text box anywhere on the page, just drag it.]

- **The event A and B can b … ol for intersection.**

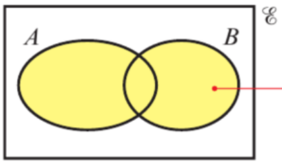


… **e sample space**.

The **intersection** of A and B is written as $A \cap B$.

If A and B are independent, $P(A \cap B) = P(A) \times P(B)$.

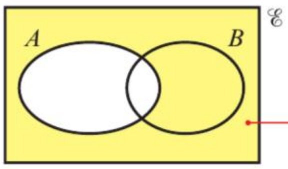
- **The event A or B can be written as $A \cup B$. The '$\cup$' symbol is the symbol for union.**



The **union** of A and B is written as $A \cup B$.

If A and B are mutually exclusive then,
$P(A \cup B) = P(A) + P(B)$.

- **The event not A can be written as A'. This is also called the complement of A.**



$P(A') = 1 - P(A)$

Events A and A' are always mutually exclusive.

The probability of an event can change depending on the outcome of a previous. Situations like this can be modelled using **conditional probability.** You use a vertical line to indicate conditional probabilities.

- **The probability that B occurs given that A has already occurred is written as $P(B|A)$.**

Similarly, $P(B|A')$ describes the probability of B occurring given that A has not occurred.

- **For independent events, $P(A|B) = P(A|B') = P(A)$, and $P(B|A) = P(B|A') = P(B)$.**

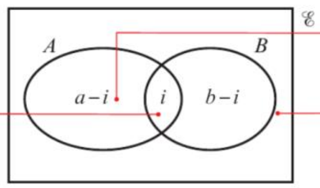
You can use this condition to determine independence.

You can solve some problems involving conditional probability by considering a **restricted sample space** of the outcomes where one event has already occurred.

There is a formula you can use for two events that links the probability of the union and the probability of the intersection.

If $P(A) = a$ and $P(B) = b$

The probability of the intersection, $P(A \cap B)$, is i.



Subtract this probability from a and b and write the probabilities on the Venn diagram as shown.

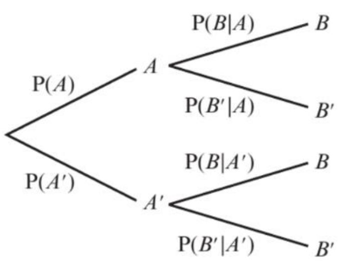
The probability of $A \cup B$ is

$$P(A \cup B) = (a - i) + (b - i) + i = a + b - i$$

Since $i = P(A \cap B)$ you can write the following **addition formula** for two events A and B:

- $\mathbf{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

Conditional probabilities can be represented on a tree diagram.



$P(A \cap B) = P(A) \times P(B|A)$

The probabilities on the second set of branches represent the conditional probabilities of B given that A has, or has not, happened.

KNOWLEDGE ORGANISER: Year 13 Statistics – Chapter 3 – The Normal Distribution

Continuous Random Variable

- can take any one of infinitely many values. The probability that a continuous random variable takes any one specific value is 0.
- A continuous random variable has a **continuous probability distribution.** This can be shown as a curve on a graph
- The **area** under a **continuous probability distribution** is equal to **1.**

The normal distribution

- has parameters, μ is the population mean and σ^2 is population variance.
- is symmetrical (mean = mode = median)
- has a bell-shaped curves with asymptotes at each end
- has a total area under the curve equal to 1
- has points of inflection at $\mu + \sigma$ and $\mu - \sigma$
- If X is a normally distributed random variable, you write $X \sim N(\mu, \sigma^2)$ where μ is the population mean and σ^2 is population variance.

The Standard Normal Distribution

It is often useful to **standardise** normally distributed random variables. By coding the data, it can be modelled by the **standard normal distribution.**

- **The standard normal distribution has mean 0 and standard deviation 1**
- **The standard normal variable is written as $Z \sim N(0, 1^2)$**

If $X \sim N(\mu, \sigma^2)$ is a normal random variable with mean, μ and standard deviation σ, then it can be coded using the formula$Z = \frac{X-\mu}{\sigma}$. The resulting z-values will be normally distributed.

Approximating a Binomial Distribution

In certain circumstances, you can use normal distribution to approximate a binomial distribution.

- **If *n* is large and *p* is close to 0.5, then the binomial distribution $X \sim B(n, p)$can be approximated by the normal distribution $N(\mu, \sigma^2)$ where**
  - $\mu = np$
  - $\sigma = \sqrt{np(1-p)}$

- If you are using a normal approximation to a binomial distribution, you need to apply a **continuity correction** when calculating probabilities

Hypothesis testing with the normal distribution

You can test hypotheses about the mean of a normally distributed random variable by looking at the mean of a sample taken from the whole population.

- **For a random sample of size *n* taken from a random variable $N(\mu, \sigma^2)$, the sample mean, $\overline{X}$, is normally distributed with $\overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$**

If you need to find a **critical region** or **critical value** for a hypotheses test for the mean of a normal distribution you can standardise your test static

- **For the sample mean of a normally distributed random variable, $\overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$, $Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$ is a normally distributed random variable with $Z \sim N(0, 1)$.**

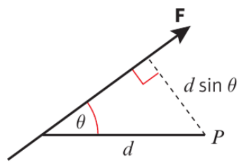
The moment of a force measures the turning effect of the force on a rigid body.

It is the **product** of the **magnitude of the force** and the **perpendicular distance** from the axis of rotation.

Moments are measured in **Nm.**

- **Clockwise moment of F about $P = |\mathbf{F}| \times d$**

- **Clockwise moment of F about $P = |\mathbf{F}| \times d \sin\theta$**



Resultant moment

- When there are several **co-planar** forces acting on a body, the turning effect around a point can be determined by choosing a positive direction (clockwise or anti-clockwise) and then finding the sum of the moments produced by each force.

- When a rigid body is in equilibrium, the resultant force in any direction is 0 N and the resultant moment about any point is 0 Nm.

- When you take moments about a given point, you can ignore the rotational effect of any forces acting at that point

Centre of Mass

- For **uniform** rods, the centre of mass is always at **the midpoint.**
- If the rod is **non-uniform** the centre of mass is **not** necessarily at the **midpoint**

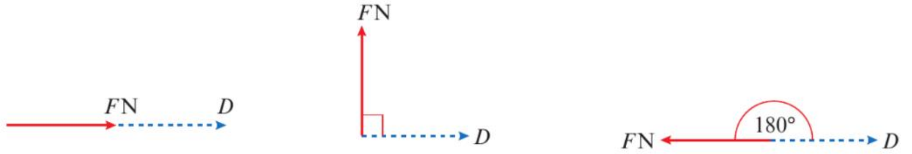
Tilting

- **When a rigid body is on the point of tilting about a pivot, the reaction at any other support (or the tension in any wore or string) is zero**

KNOWLEDGE ORGANISER: Year 13 Mechanics – Chapter 5 – Forces & Friction

- **If a force is applied at an angles to the direction of motion you can resolve parallel and perpendicular to the plane**

- **The component of a force of magnitude F in a certain direction is $F\cos\theta$, where θ is the size of the angle between the force and the direction.**



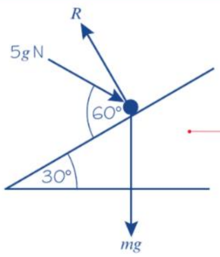
If F acts in the direction D, then the component of F in that direction is $F\cos 0° = F \times 1 = F$

If F acts at the right angles to D, then the component of F in that direction is $F\cos 90° = F \times 0 = 0$

If F acts in the opposite direction to D, then the component of F in that direction is $F\cos 180° = F \times -1 = -F$

Inclined planes

- **To solve problems involving inclined planes, it is easier to resolve parallel and perpendicular to the plane**



Draw a diagram to show all the forces acting on the particle.

R(↗), $5g\cos 60° - mg\sin 30° = 0.5m$

$2.5g - 0.5mg = 0.5m$

$2.5g = 0.5m + 0.5mg$

$5g = m + mg$

$5g = m(1 + g)$

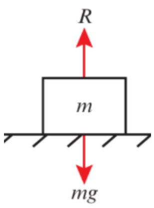
Resolve up the slope, in the direction of the acceleration, and write an equation of motion for the particle.

$m = \left(\frac{5g}{1+g}\right)$ kg as required

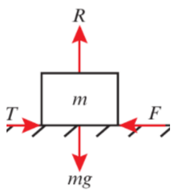
You need to find the mass of the particle in terms of g, so you don't need to use $g = 9.8\text{ m s}^{-2}$ in your working.

Friction is a force that opposes motion between two rough surfaces.

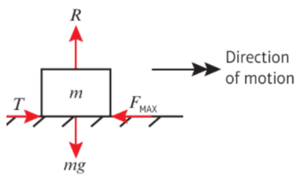
This block is stationary. There is no horizontal force being applied, so there is no tendency for the block to move. There is no frictional force acting on the block



This block is also stationary. There is a horizontal force being applied that is not sufficient to move the block. There is a tendency for the block to move, but it doesn't because the force of friction is equal and opposite to the force being applied



As the applied force increases, the force of friction increases to prevent the block from moving. If the magnitude of the applied force exceeds a certain **maximum** or **limiting value**, the block will move. While the block moves, the force of friction will remain constant at its maximum value.



The limiting value of the friction depends on two things:

- The normal reaction R between the two surface in contact
- The roughness of the two surfaces in contact.

The maximum or limiting value of the friction between two surfaces, F_{max}, is given by

$$F_{max} = \mu R$$

Where, μ is the coefficient of friction and R is the normal reaction between the two surfaces.

KNOWLEDGE ORGANISER: Year 13 Mechanics – Chapter 6 – Projectiles

Horizontal Projection

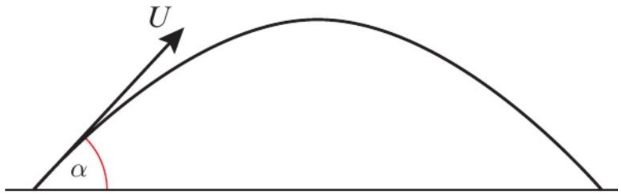
The motion of a projectile can be modelled as a particle being acted on by a single force, gravity. In this model ignore the effects of air resistance and any rotational movement on the particle

- **The horizontal motion of a projectile is modelled as having constant velocity ($a = 0$). Use the formula $s = vt$**
- **The vertical motion of a projectile is modelled as having constant acceleration due to gravity ($a = g$)**

Horizontal and Vertical Components

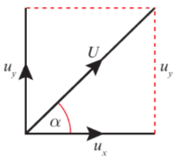
Suppose a particle is projected with initial velocity U, at an angle α above the horizontal. The angle α is called the **angle of projection**.

You can **resolve** the velocity into **components** that act horizontally and vertically:



$\cos\alpha = \frac{u_x}{U}$ so $u_x = U\cos\alpha$

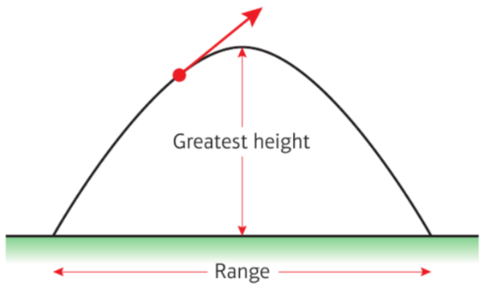
$\sin\alpha = \frac{u_y}{U}$ so $u_y = U\sin\alpha$



When a particle is projected with the initial velocity U, at an angle α above the horizontal:

- **The horizontal component of the initial velocity is $U\cos\alpha$**
- **The vertical component of the initial velocity is $U\sin\alpha$**

Projection at any Angle



The distance from the point from which the particle was projected to the point where it strikes the horizontal plane is called the **range**

The time the particle takes to move from its point of projection to the point where it strikes the horizontal plane is called the **time of flight** of the projectile.

- **A projectile reaches its point if greatest height when the vertical component of its velocity is equal to 0**

For a particle which is projected from a point on a horizontal plane with an initial velocity U at an angle α above the horizontal, and that moves freely under gravity:

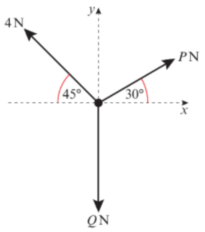
- **Time of flight** $= \frac{2U\sin\alpha}{g}$
- **Time to reach greatest height** $= \frac{U\sin\alpha}{g}$
- **Range on horizontal plane** $= \frac{U^2\sin 2\alpha}{g}$
- **Equation of trajectory:** $y = x\tan\alpha - gx^2\frac{(1+\tan^2\alpha)}{2U^2}$

KNOWLEDGE ORGANISER: Year 13 Mechanics – Chapter 7 – Applications of Forces

Static Particles

A particle or rigid body is in static equilibrium if it is at rest and the resultant force acting on the particle is zero. To solve problems in statics you should:

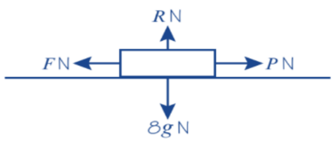
- Draw a diagram showing clearly the forces acting on the particle
- Resolve the forces into horizontal and vertical components or, if the particle is on an inclined plane, into components parallel and perpendicular to the plane.
- Set the sum of the components in each direction equal to zero.
- Solve the resulting equations to find the unknown force(s)



Friction & Static Particles

When a body is in static equilibrium under the action of a number of forces, including friction, you need to consider whether the body is on the point of moving or not.

- **The maximum value of the frictional force $F_{max} = \mu R$ is reached when the body you are considering is on the point of moving. The body is then said to be in limiting equilibrium**
- **In general, the force of friction F is such that $F \leq \mu R$, and the direction of the frictional force is opposite to the direction in which the body would move if the frictional force were absent.**



This is an example of limiting equilibrium.

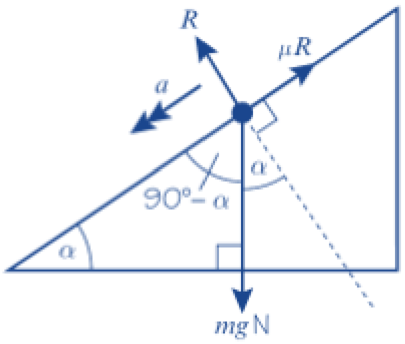
Static Rigid Bodies

If you need to consider the rotational forces, acting on an object you can model it as a **rigid body.**

- **For a rigid body in static equilibrium**
  - **The body is stationary**
  - **The resultant force in any direction is zero**
  - **The resultant moment is zero**

Dynamics & Inclined Planes

When a particle is moving along a rough plane, the force of friction is equal to μR, and act parallel to the plane to oppose the direction of motion.



KNOWLEDGE ORGANISER: Year 13 Mechanics – Chapter 8 – Further Kinematics

Vectors in Kinematics

Two-dimensional vectors can be used to describe motion in a plane

- **If a particle starts from the point with position vector r_0 and moves with constant velocity, v, then its displacement from its initial position vector r is given by $r = r_o + vt$**
- **Unless otherwise told, assume that *i* and *j* are unit vectors due east and north respectively.**

Vector Methods with Projectiles

Projectile motion is motion in a vertical plane with constant acceleration. Hence, you can analyse it using the vector equations of motion. When using vectors with projectile questions consider *i* and *j* to be the unit vectors horizontally and vertically, unless told otherwise.

You can solve questions involving constant acceleration in two dimensions using the vector equations of motion.

- **For an object moving in a plane with constant acceleration:**
  - $\mathbf{v} = \mathbf{u} + \mathbf{a}t$
  - $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

  where
  - **u is the initial velocity**
  - **a is the acceleration**
  - **v is the velocity at time *t***
  - **r is the displacement at time *t***

Variable Acceleration in One Dimension

When a body experiences variable acceleration, the acceleration can be modelled ad a function of time. Calculus can be used to describe the relationship between displacement, velocity and acceleration.

Differentiating Vectors:

If $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j}$, then $\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = \dot{\boldsymbol{r}} = \dot{x}\boldsymbol{i} + \dot{y}\boldsymbol{j}$

and $\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} = \frac{d^2\boldsymbol{r}}{dt^2} = \ddot{\boldsymbol{r}} = \ddot{x}\boldsymbol{i} + \ddot{y}\boldsymbol{j}$

Integrating Vectors:

$\boldsymbol{v} = \int \boldsymbol{a}\ dt$ and $\boldsymbol{r} = \int \boldsymbol{v}\ dt$

- Integrate vectors in the form $f(t)\boldsymbol{i} + g(t)\boldsymbol{j}$ by integrating each function separately.
- When a vector is integrated the constant of integration will be in the form $\boldsymbol{c} = p\boldsymbol{i} + q\boldsymbol{j}$