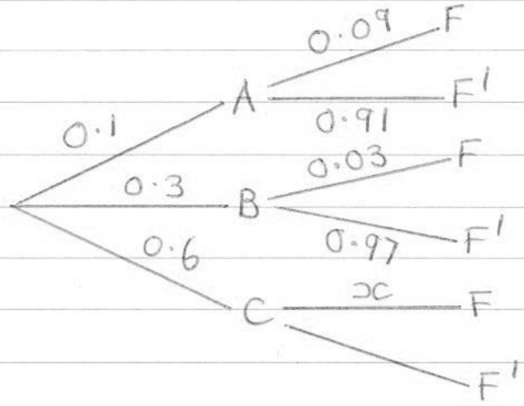
2018 AS Applied – Solutions

(S) 1/ (a) Positive correlation

(b) For each extra point, pay increases by £4.50. [context]

(c) Some jobs would have negative pay.
eg. for 10 points,
$y = 4.5(10) - 47 = -£2.$

2/ (a)



$P(\text{faulty}) = 0.06$

$0.1(0.09) + 0.3(0.03) + 0.6(x) = 0.06$

$x = 0.07$

7% are faulty

(b) The probability of being faulty is different for each supplier.
A component bought from supplier B is less likely to be faulty than a component bought from either of suppliers A or C.
Therefore the choice of supplier affects the probability of the component being faulty, so the two events are not independent.
[Could also justify independence using calculations.]

3, (a) X = "no. wins out of 15"
$X \sim B(15, \frac{1}{3})$

(i) $P(X=2) = \binom{15}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{13}$
$= 0.05994602934$
$= \underline{\underline{0.0599}}$ (3sf)

(ii) $P(X>5) = \cancel{B}\, 1 - P(X \leq 5)$
$= 1 - 0.61838256$ [calculator]
$= 0.38161744$
$= \underline{\underline{0.382}}$ (3sf)

(b) X = "no. wins out of 32"

$H_0: p = \frac{1}{3}$
$H_1: p > \frac{1}{3}$ ["win more than"]

If H_0, $X \sim B(32, \frac{1}{3})$

$P(X \geq 16)$ [16 or less likely]
$= 1 - P(X \leq 15)$
$= 1 - 0.96235007$
$= 0.03764993$

Since $0.0376 < 0.05$, we reject H_0 and conclude that there is evidence at the 5% significance level to support Naasir's claim.

4/ (a) $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{184}{18} = 10.\dot{2}$

$= \underline{\underline{10.2 \text{ knots}}}$ (3sf)

(b) $\sigma = \sqrt{\frac{\Sigma x^2 f}{n} - (\bar{x})^2}$

$= \sqrt{\frac{2062}{18} - (10.\dot{2})^2}$

$= 3.172022761$

$= \underline{\underline{3.17 \text{ knots}}}$ (3sf)

(c) This likely to be October [or September] since Camborne is windier in the Autumn and October is the last month of the year that is included in the study.

(d)(i) The 'x' represents an <u>outlier</u>. [<u>not</u> 'anomaly']

(ii) We expect the mean to be similar to the median. Since the mean is affected by the outlier, but the median is not, we expect a mean > 7.

The IQR for Y is not the smallest, and the range for Y is one of the largest, so we expect a high value for σ.

Therefore it is most likely to be B. [or C, or D]

5, (a) $P(X=r) = P(X=r+2)$ $r=1,2$

$\Rightarrow P(X=1) = P(X=3)$

$P(X=2) = P(X=4) = 0.35$

$P(X=1) = P(X=3) = \frac{1}{2}(1-(0.35 \times 2))$

$= 0.15$

x	1	2	3	4
$P(x)$	0.15	0.35	0.15	0.35

[or function]

(b) F = "no. of 4's out of 60"

$F \sim B(60, 0.35)$

$P(F > 30) = 1 - P(F \leq 30)$

$= 1 - 0.99411010$ [calculator]

$= 0.0058899$

$= \underline{\underline{0.00589}}$ (3sf)

(c) $Y = \frac{12}{X}$

X	1	2	3	4
Y	12	6	4	3
Y−X	11	4	1	−1

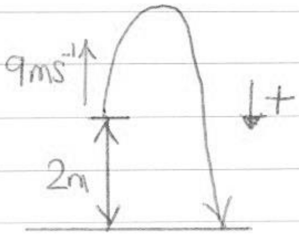
$P(Y-X \leq 4) = P(X=2) + P(X=3) + P(X=4)$

$= 0.35 + 0.15 + 0.35$

$= \underline{\underline{0.85}}$

[This can also be worked out using algebra.]

(m) 6/



$t = ?$

$s = 2$

$u = -9$

$a = 10$

$s = ut + \frac{1}{2}at^2$

$2 = -9t + \frac{1}{2}(10)t^2$

$5t^2 - 9t - 2 = 0$

$(5t+1)(t-2) = 0$

$t = -\frac{1}{5}$ or 2

$\underline{\underline{T = 2}}$

7/ (a)(i) acceleration $= \dfrac{\text{change in velocity}}{\text{time}}$

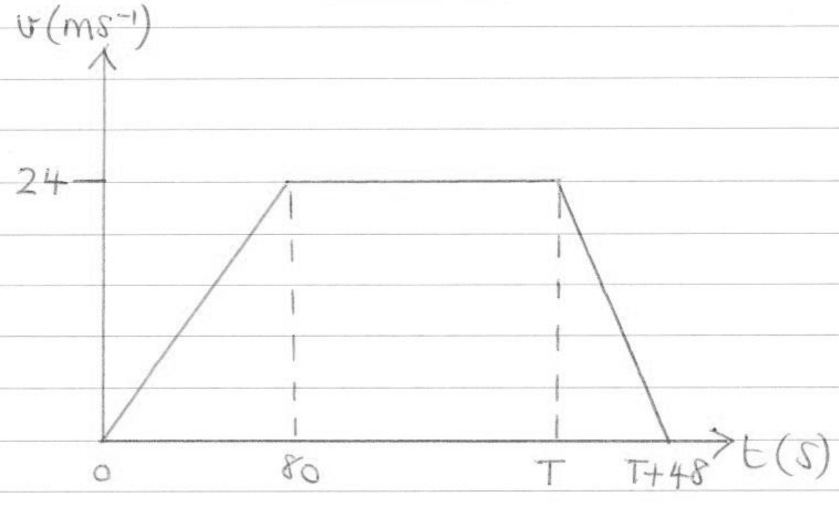
$0.3 = \dfrac{v - 0}{80}$ $\quad \left[a = \dfrac{v-u}{t} \right]$

$v = \underline{\underline{24\ ms^{-1}}}$

(ii) $-0.5 = \dfrac{0 - 24}{t}$

$t = \underline{\underline{48\ secs}}$

(iii)



[The right shape, with ruled lines and labelled axes, gained this mark]

$\left[\frac{1}{2}(a+b)h\right]$

7, (b) $\frac{1}{2}\left[(T-80)+(T+48)\right](24) = 4800$

$\frac{1}{2}(2T-32)(24) = 4800$

$T = 216$

total time $= T+48 = 216+48 = \underline{\underline{264\ s}}$

(c) The model could be improved using a variable (rather than constant) rate of acceleration and deceleration.

8, (a) $x = \frac{1}{2}t^2(t^2-2t+1) = \frac{1}{2}t^4 - t^3 + \frac{1}{2}t^2$

$v = \frac{dx}{dt} = 2t^3 - 3t^2 + t$

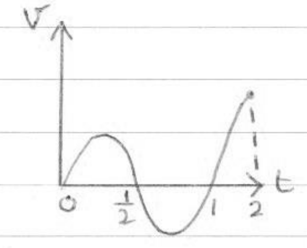
At rest $(v=0)$, $2t^3-3t^2+t = 0$

$t(2t^2-3t+1) = 0$

$t(2t-1)(t-1) = 0$

$t = \underline{\underline{0, \frac{1}{2} \text{ or } 1 \text{ secs}}}$

(b)



At $t=0$, $x=0$

At $t=\frac{1}{2}$, $x = \frac{1}{2}\left(\frac{1}{2}\right)^2\left(\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)+1\right)$

$= \cancel{0.28125}\ 0.03125$

At $t=1$, $x = \frac{1}{2}(1^2)(1^2-2(1)+1)$

$= 0$

At $t=2$, $x = \frac{1}{2}(2^2)(2^2-2(2)+1)$

$= 2$

total dist. $= \overset{0.03125}{\cancel{0.28125}} + \overset{0.03125}{\cancel{0.28125}} + 2$

$= 2.\overset{0}{\cancel{5}}625 = \underline{\underline{2.\overset{0}{\cancel{5}}6\ m}}$ (3sf)

(c) $x<0$, $\frac{1}{2}t^2(t^2-2t+1) < 0$

$t^2-2t+1 < 0$ since $\frac{1}{2}t^2 \geq 0$

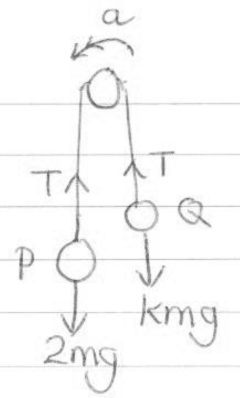
$(t-1)^2 < 0$ invalid.

$\therefore$ P will never move along the negative x-axis.

$\square$

9/ (a) For P, $F_R = ma$

$$2mg - T = 2m\left(\frac{5g}{7}\right)$$
$$T = \frac{4}{7}mg$$



(b) The string is <u>inextensible</u>,
so the tension in the string is the same
at all points along its length,
so the acceleration at Q is the same magnitude as
the acceleration at connected particle P.

(c) For Q, $F_R = ma$

$$T - kmg = km\left(\frac{5g}{7}\right)$$
$$\frac{4}{7}mg - kmg = \frac{5}{7}kmg$$
$$\frac{4}{7} - k = \frac{5}{7}k$$
$$\frac{4}{7} = \frac{12}{7}k$$
$$4 = 12k$$
$$k = \frac{1}{3}$$

(d) eg. In practice there would be friction
at the pulley

[Use any of the modelling assumptions
given in the question.]