Year 10 Maths

Unit 1: Powers and roots

The **reciprocal** of a number is 1 divided by the number.
For a fraction, this has the effect of inverting it (turning it upside down).
e.g. the reciprocal of $\frac{5}{3}$ is $\frac{3}{5}$

power of -1...

EXAMPLE:
Evaluate (find the value)

(a) $\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$

This is the reciprocal

(b) $5^{-1} = \frac{1}{5}$

5 can be written as $\frac{5}{1}$ so its reciprocal is $\frac{1}{5}$

(c) $\left(\frac{1}{7}\right)^{-1} = \frac{7}{1} = 7$

other negative powers

EXAMPLE:
Evaluate $\left(\frac{4}{5}\right)^{-2}$

$= \left(\frac{5}{4}\right)^{2}$

The negative part of the power creates a reciprocal

$= \frac{25}{16}$

Then use the number part of the power

more negative powers

EXAMPLE:
Write in the form 3^n.

"in the form" tells us how the answer should look

(a) $81 = 3^4$

(b) $\frac{1}{3} = 3^{-1}$

(c) $\frac{1}{9} = \frac{1}{3^2} = 3^{-2}$

EXAMPLE:
Write $7n^{-3}$ as a fraction.

$= 7 \times n^{-3}$

$= 7 \times \frac{1}{n^3}$

$= \frac{7}{n^3}$

EXAMPLE:
Write $\frac{1}{m^5}$ using a power

$= m^{-5}$

EXAMPLE:
Write $\frac{1}{4m^5}$ using a power

$= \frac{1}{4}m^{-5}$

The power doesn't apply to the 4

×/÷ with indices...

EXAMPLE:
Simplify $p^5 \times p^7$

$= p^{12}$

*When we multiply with the same **base**, we add the powers*

EXAMPLE:
Simplify $7m^6 \times 3m^{-2}$

$= 21m^4$

Multiply the numbers before adding powers

EXAMPLE:
Simplify $\frac{3x^7}{6x^2}$

$= \frac{1}{2}x^5$

$\frac{3}{6}$ simplifies to $\frac{1}{2}$

When we divide with the same base, we subtract the powers

power of power...

For a power of a power, we *multiply* the indices.

EXAMPLE:
Simplify $(x^6)^5$

$= x^{30}$

EXAMPLE:
Simplify $(4x^6)^3$

$= 64x^{18}$

4 needs to be cubed too

EXAMPLE:
Simplify $(5a^4b^7)^2$

$= 25a^8b^{14}$

square the 5, then the a^4, then the b^7

power 0...

Anything to **power 0** is 1.

EXAMPLE:
Simplify k^0

$= 1$

EXAMPLE:
Simplify $5k^0$

$= 5 \times 1 = 5$

$k^0 = 1$ then we need to multiply by 5

EXAMPLE:
Simplify $(5k)^0$

$= 1$

combining rules...

EXAMPLE:
Simplify $\frac{8h^4 \times 2h^5}{2h^{-3}}$

$= \frac{16h^9}{2h^{-3}}$

Simplify the numerator

$= 8h^{12}$

16 ÷ 2 = 8 and 9 - -3 = 12

EXAMPLE: Simplify $(3t^5 \times t)^2 \times t^{-12}$

$= (3t^6)^2 \times t^{-12}$

$= 9t^{12} \times t^{-12}$

$= 9t^0 = 9 \times 1 = 9$

Anything to power 0...

In a fraction power, the denominator is a root and the numerator is a power.
e.g.

$$16^{\frac{5}{2}} = (\sqrt{16})^5 \text{ or } \sqrt{16^5}$$

$$64^{\frac{2}{3}} = (\sqrt[3]{64})^2 \text{ or } \sqrt[3]{64^2}$$

fraction powers...

EXAMPLE:

Evaluate $125^{\frac{2}{3}}$

$$= \left(\sqrt[3]{125}\right)^2$$

$$= (5)^2$$

$$= 25$$

Easiest to start with the root

Then the power

EXAMPLE:

Evaluate $16^{\frac{1}{4}}$

$$= \sqrt[4]{16}$$

$$= 2$$

Just the root this time, as the numerator is 1

$2^4 = 16$

A **rational** number can be written as a fraction or integer.

An **irrational** number can't. Its decimal representation goes on forever, without recurring. (π is irrational.)

A **surd** is an *irrational* number which involves a root.
e.g. $\sqrt{2}$, $5 - \sqrt{7}$, $\sqrt[3]{10}$

Not all roots are surds.
e.g. $\sqrt{9}$ isn't, because it equals 3, which is rational.

negative too...

EXAMPLE:

Evaluate $\left(\frac{16}{25}\right)^{-\frac{3}{2}}$

$$= \left(\frac{25}{16}\right)^{\frac{3}{2}}$$

$$= \frac{\left(\sqrt{25}\right)^3}{\left(\sqrt{16}\right)^3}$$

$$= \frac{5^3}{4^3}$$

$$= \frac{125}{64}$$

The negative power is a reciprocal

Then apply the fraction power

add/subtract surds...

When adding and subtracting, we collect terms in the same surd.

EXAMPLE:
Simplify

(a) $7\sqrt{5} + 2\sqrt{5} - \sqrt{5}$

$$= 8\sqrt{5}$$

$7 + 2 - 1 = 8$

(b) $9\sqrt{2} + 6\sqrt{7} - 4\sqrt{2}$

$$= 5\sqrt{2} + 6\sqrt{7}$$

Only the like terms can be collected

multiply/divide...

EXAMPLE: Write as a single surd: $\sqrt{8} \times \sqrt{5}$

$= \sqrt{8 \times 5} \; = \sqrt{40}$

This doesn't work with adding: $\sqrt{8} + \sqrt{5}$ is not equal to $\sqrt{13}$

EXAMPLE: Simplify $3\sqrt{5} \times 4\sqrt{2}$

$= 12\sqrt{10}$

Multiply 3 & 4, then the surds

EXAMPLE: Simplify $\dfrac{\sqrt{21}}{\sqrt{7}}$

$= \sqrt{21 \div 7}$
$= \sqrt{3}$

surd brackets...

EXAMPLE: Expand $\sqrt{3}(2 + \sqrt{5})$

	2	$+\sqrt{5}$
$\sqrt{3}$	$2\sqrt{3}$	$\sqrt{15}$

$= 2\sqrt{3} + \sqrt{15}$

A grid is a good way to expand the brackets

EXAMPLE: Expand and simplify $(2 + \sqrt{3})(6 - \sqrt{3})$

	2	$+\sqrt{3}$
6	12	$6\sqrt{3}$
$-\sqrt{3}$	$-2\sqrt{3}$	-3

$= 12 + 6\sqrt{3} - 2\sqrt{3} - 3$
$= 9 + 4\sqrt{3}$

Collect the integers and surds separately

$\sqrt{3} \times \sqrt{3}$ is just 3

simplifying surds...

A surd is simplified if the number under the root is as small as possible.

EXAMPLE: Simplify $\sqrt{75}$

$= \sqrt{25} \times \sqrt{3}$
$= 5\sqrt{3}$

*Find a **square number** that's a factor of 75*

EXAMPLE: Simplify $6\sqrt{27}$

$= 6 \times \sqrt{9} \times \sqrt{3}$
$= 6 \times 3 \times \sqrt{3}$
$= 18\sqrt{3}$

EXAMPLE: Simplify $\sqrt{180}$

$= \sqrt{9} \times \sqrt{20}$
$= 3 \times \sqrt{20}$
$= 3 \times \sqrt{4} \times \sqrt{5}$
$= 3 \times 2 \times \sqrt{5}$
$= 6\sqrt{5}$

Sometimes we can simplify further, using another square number

We consider a fraction simplified if it doesn't have a surd in the denominator.

The process of getting rid of a surd from the denominator is called **rationalising**.

1-term denominator...

EXAMPLE:
Simplify

(a) $\frac{5}{\sqrt{2}}$

$$= \frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

Multiply the numerator and denominator by the surd

$$= \frac{5\sqrt{2}}{2}$$

$\sqrt{2} \times \sqrt{2} = 2$

(b) $\frac{5}{3\sqrt{2}}$

$$= \frac{5}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

Multiply by $\sqrt{2}$ not by $3\sqrt{2}$

$$= \frac{5\sqrt{2}}{6}$$

$3\sqrt{2} \times \sqrt{2}$
$= 3 \times 2 = 6$

(c) $\frac{12}{5\sqrt{3}}$

$$= \frac{12}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

Multiply by $\sqrt{2}$ not by $3\sqrt{2}$

$$= \frac{12\sqrt{3}}{15}$$

$5\sqrt{3} \times \sqrt{3}$
$= 5 \times 3 = 15$

$$= \frac{4\sqrt{3}}{5}$$

$\frac{12}{15}$ *can be simplified to* $\frac{4}{5}$

2-term denominator...

EXAMPLE: Simplify
$\frac{5}{3+\sqrt{2}}$

$$= \frac{5}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$$

*$3-\sqrt{2}$ is called the **conjugate** of $3+\sqrt{2}$*

$$= \frac{5(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})}$$

	3	$+\sqrt{2}$
3	9	$3\sqrt{2}$
$-\sqrt{2}$	$-3\sqrt{2}$	-2

A grid can help with the double brackets

$$= \frac{15-5\sqrt{2})}{9+3\sqrt{2}-3\sqrt{2}-2}$$

$$= \frac{15-5\sqrt{2}}{7}$$

$+3\sqrt{2}-3\sqrt{2}=0$
and $9-2=7$