Year 10 Maths

Unit 3: Brackets

$$5x + 2x \equiv 7x$$

This is an **identity**. It is a statement that is always true, because the two halves are the same, just written differently. We often use this special equals symbol for an identity: $\equiv$

expanding brackets...

The word **expand** means to 'get rid of' any brackets, by multiplying.

EXAMPLE:
Expand $4y(8-2y)$

Draw a grid and put the terms from the question around the outside

	8	$-2y$
$4y$	$32y$	$-8y^2$

Then fill in the middle, by multiplying

$$= 32y - 8y^2$$

EXAMPLE:
Expand and simplify $(2p+3)(p-5)$

One bracket on each side of the grid

	$2p$	$+3$
p	$2p^2$	$3p$
-5	$-10p$	-15

$$= 2p^2 + 3p - 10p - 15$$

$$= 2p^2 - 7p - 15$$

Simplify the like terms:
$3p - 10p = -7p$
because $3 - 10 = -7$

proving identities...

EXAMPLE: Show that
$$(x-3)(x-2) \equiv 6 + x(x-5)$$

Expand the brackets

	x	-2
x	x^2	$-2x$
-3	$-3x$	6

	x	-5
x	x^2	$-5x$

Simplify each side in turn, showing all the steps

LHS $= x^2 - 2x - 3x + 6$
$\quad\; = x^2 - 5x + 6$

RHS $= 6 + x^2 - 5x$
$\quad\; = x^2 - 5x + 6$

Finish with a conclusion, like this

$\therefore$ LHS = RHS

The symbol $\therefore$ is a quick way of writing 'therefore'

another identity...

EXAMPLE: Prove that
$(x+4)(3-x)-2(x+6)$
$= -x^2-3x$
for all values of x.

This time the right side is already simplified

	x	$+4$
3	$3x$	12
$-x$	$-x^2$	$-4x$

	x	$+6$
-2	$-2x$	-12

So we can start with the left side and end up with the right side

$$\text{LHS} = 3x+12-x^2-4x-2x-12$$
$$= -x^2-3x$$
$$= \text{RHS}$$

∴ LHS = RHS

Remember the conclusion

triple brackets...

EXAMPLE:
Expand and simplify
$(x+3)(x+2)(2x-1)$

Expand any two of the brackets

	x	$+2$
$2x$	$2x^2$	$4x$
-1	$-x$	-2

$$2x^2+4x-x-2$$
$$= 2x^2+3x-2$$

$$(x+3)(2x^2+3x-2)$$

Combine with the third bracket

	$2x^2$	$+3x$	$+2$
x	$2x^3$	$3x^2$	$2x$
$+3$	$6x^2$	$9x$	6

Now multiply it all out

$$= 2x^3+3x^2+2x+6x^2+9x+6$$

Finally, collect the x^2 and x^3 terms

$$= 2x^3+9x^2+11x+6$$

factorise single...

EXAMPLE:
Factorise $10-15p$

Write the expression in the grid

Find the highest common factor

	2	$-3p$
5	10	$-15p$

$$= 5(2-3p)$$

EXAMPLE:
Factorise $8x^2-12x$

$4x$ is the highest common factor

	$2x$	-3
$4x$	$8x^2$	$-12x$

$$= 4x(2x-3)$$

algebraic fractions...

We simplify an algebraic fraction by factorising both the numerator and denominator

EXAMPLE:
Simplify $\frac{5x+10}{3x+6}$

Factorise the numerator and denominator

	x	2
5	$5x$	$+10$

	x	2
3	$3x$	$+6$

$$= \frac{5(x+2)}{3(x+2)}$$

$$= \frac{5}{3}$$

Divide the numerator and denominator by $(x+2)$

EXAMPLE:
Simplify $\frac{2x^2-6x}{x^2+5x}$

	x	-3
$2x$	$2x^2$	$-6x$

	x	5
x	x^2	$+5x$

$$= \frac{2x(x-3)}{x(x+5)}$$

$$= \frac{2(x-3)}{x+5}$$

This time we can divide both by x

A **quadratic** expression has positive powers. The highest power is 2.

Factorising x^2...

EXAMPLE:
Factorise $x^2+7x+12$

+ ×

List all the pairs that multiply to make 12. Find the pair that also adds to 7

1, 12
<u>3, 4</u>
2, 6

The brackets need x & x to make x^2. They also have our chosen pair of numbers: 3, 4

$$= (x+3)(x+4)$$

EXAMPLE:
Factorise $x^2-3x-10$

+ ×

5, -2
<u>-5, 2</u>
10, -1
-10, 1

$$= (x-5)(x+2)$$

Factorising ax^2...

For $2x^2$, $3x^2$, etc. we can use a grid to help factorise them.

EXAMPLE: Factorise $2x^2 + 7x + 3$

2 x 3 = 6

List all the pairs that multiply to make 6. Find the pair that also adds to 7

x, 6x ← 1, 6
2, 3

Put the x and 6x in the grid, along with the $2x^2$ and 3 from the question.

	$2x$	1
x	$2x^2$	x
3	$6x$	3

Then factorise the rows and columns

$= (2x + 1)(x + 3)$

EXAMPLE: Factorise $4x^2 - 16x + 15$

4 x 15 = 60

We're looking for two numbers that multiply to make 60 and add to -16. This must be two negatives

-60, -1   -15, -4
-30, -2   -12, -5
-20, -3   -10, -6

-10x, -6x

List all the pairs of two negatives that multiply to make 60.

Find the pair that also adds to -16

Put the -10x and -6x in the grid, along with the $4x^2$ and 15 from the question.

	$2x$	-5
$2x$	$4x^2$	$-10x$
-3	$-6x$	15

$= (2x - 5)(2x - 3)$

dots...

An expression like $x^2 - 25$ or $36 - 4y^2$ is a **difference of two squares**.
(Notice that 25, 36 and 4 are **square numbers**.)
These factorise into brackets that are identical apart from having opposite signs.

EXAMPLE: Factorise $x^2 - 49$

$$\sqrt{x^2} = x$$
$$\sqrt{49} = 7$$

$$= (x + 7)(x - 7)$$

EXAMPLE: Factorise $49 - x^2$

$$\sqrt{49} = 7$$
$$\sqrt{x^2} = x$$

$$= (7 + x)(7 - x)$$

EXAMPLE: Factorise $9x^2 - 25$

$$\sqrt{9x^2} = 3x$$
$$\sqrt{25} = 5$$

$$= (3x + 5)(3x - 5)$$