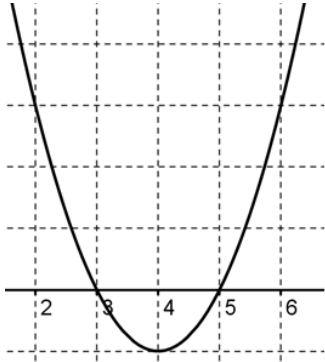




In a **quadratic** equation, the highest power of x is 2.

The graph of a quadratic function makes this U-shaped curve, called a **parabola**.

The **roots** of a quadratic function are the values of x where the graph meets the x -axis. We can identify these points by solving a quadratic equation.

For the graph on the left, the roots are 3 and 5.

Factorising x^2 ...

EXAMPLE:

Factorise $x^2 + 7x + 12$

+

x

1, 12

3, 4

2, 6

$$= (x + 3)(x + 4)$$

EXAMPLE:

Factorise $x^2 - 3x - 10$

+

x

5, -2

-5, 2

10, -1

-10, 1

$$= (x - 5)(x + 2)$$

List all the pairs that multiply to make 12. Find the pair that also adds to 7

First factorise.
(We need two negative numbers to multiply to make +10)

The brackets need x & x to make x^2 . They also have our chosen pair of numbers: 3, 4

The solutions are the values of x which make the brackets 0.

factorise & solve...

EXAMPLE:

Solve by factorising:

$$x^2 - 7x + 10 = 0$$

+

x

-1, -10

-2, -5

$$(x - 2)(x - 5) = 0$$

$$x - 2 = 0 \text{ or } x - 5 = 0$$

$$x = 2 \text{ or } x = 5$$

Hence identify the roots of the function:

$$y = x^2 - 7x + 10$$

$$x = 2 \text{ and } x = 5$$

The solutions are the roots: where the graph $y = x^2 - 7x + 10$ would cross the x -axis.

rearranging first...

To be able to solve the quadratic equation, it needs to ' $= 0$ ' before we factorise it.

EXAMPLE:

Solve by factorising:

$$x^2 - 7x - 12 = 6$$

$$\begin{array}{l} -6 \left(x^2 - 7x - 12 = 6 \right) -6 \\ \quad \quad \quad x^2 - 7x - 18 = 0 \end{array}$$

+ **X**

1, -18

-1, 18

2, -9

-2, 9

$$(x + 2)(x - 9) = 0$$

$$x + 2 = 0 \text{ or } x - 9 = 0$$

$$x = -2 \text{ or } x = 9$$

Make equation equal 0

Now factorise and solve

Identify the values of a , b & c

Substitute into the quadratic formula. Use brackets on the calculator for negative values.

a useful trick...

EXAMPLE:

Solve by factorising:

$$2x^2 + 14x + 20 = 0$$

$$\begin{array}{l} \div 2 \left(2x^2 + 14x + 20 = 0 \right) \div 2 \\ \quad \quad \quad x^2 + 7x + 10 = 0 \end{array}$$

⋮

$$(x + 5)(x + 2) = 0$$

$$x + 5 = 0 \text{ or } x + 2 = 0$$

$$x = -5 \text{ or } x = -2$$

Notice that every term is divisible by 2

Divide by 2 to get a simple x^2

Replace the ' $\pm$ ' with '+' then '-' to get two solutions. Write these in full.

Then solve by factorising, as normal

We can solve *any* quadratic equation using the **quadratic formula**, including quadratics that can't be factorised.

In general, for the equation:

$$ax^2 + bx + c = 0$$

the solutions are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

quadratic formula...

EXAMPLE:

Solve $3x^2 - 2x - 7 = 0$

Give solutions correct to 3 significant figures.

$$\begin{array}{c} 3x^2 - 2x - 7 = 0 \\ \uparrow \quad \quad \uparrow \quad \quad \uparrow \\ a = 3 \quad b = -2 \quad c = -7 \end{array}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 3 \times (-7)}}{2 \times 3}$$

$$= 1.896805253 \text{ or } -1.230138587$$

$$= 1.90 \text{ or } -1.23$$

Round the solutions as we were asked to in the question

Solutions can be checked by substituting them into the original equation

$$x + y = 10$$

This equation has an infinite number of solutions (e.g. $x = 3.5$ and $y = 6.5$).

$$x + y = 10$$

$$2x - y = 14$$

However, when we combine it with another equation, there is only one pair of values that are solutions to both equations. ($x = 8$ and $y = 2$)

We call these pairs of equations, **simultaneous equations**.

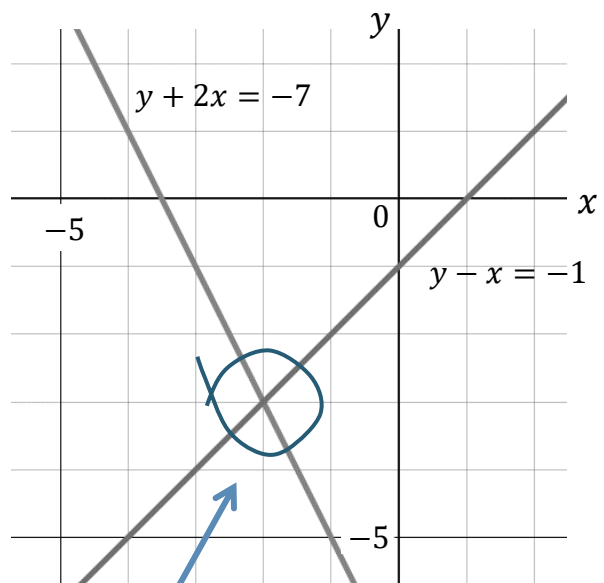
solving graphically...

EXAMPLE:

Use the graph to estimate the solutions to the simultaneous equations:

$$y + 2x = -7$$

$$y - x = -1$$



$$x = -2, y = -3$$

Notice that the equations match the lines on the graph

The solutions are the point where the two graphs intersect

solve by adding...

EXAMPLE: Solve

$$x + y = 10$$

$$2x - y = 14$$

ADD,

$$\begin{array}{r} 3x = 24 \\ \div 3 \quad \left(\right) \div 3 \\ x = 8 \end{array}$$

$$x + y = 10$$

$$8 + y = 10$$

$$y = 2$$

Adding the two equations eliminates y :

$$x + 2x = 3x$$

$$y + -y = 0$$

$$10 + 14 = 28$$

Subtracting eliminates x
 $3y - -y = 4y$

Substitute to find the other value

or by subtracting...

EXAMPLE: Solve

$$2x + 3y = 25$$

$$2x - y = 13$$

SUBTRACT,

$$\begin{array}{r} 4y = 12 \\ \div 4 \quad \left(\right) \div 4 \\ y = 3 \end{array}$$

$$2x - y = 13$$

$$2x - 3 = 13$$

$$x = 8$$

add or subtract?

We can decide whether to add or subtract using the 'STOP' rule.



SAME – TAKE
OPPOSITE – PLUS

$$\begin{aligned} 3x - 6y &= 2 \\ 5x - 6y &= 7 \end{aligned}$$

-6y and -6y are both negative.
We would subtract these equations.
(SAME – TAKE)

$$\begin{aligned} 2x + 3y &= 8 \\ 2x - 4y &= -6 \end{aligned}$$

2x and 2x are both positive.
We would subtract these equations.
(SAME – TAKE)

$$\begin{aligned} 5x + 2y &= 11 \\ 7x - 2y &= 1 \end{aligned}$$

+2y and +2y have opposite signs.
We would add these equations.
(OPPOSITE – PLUS)

scaling up...

To eliminate a variable,
we need the same
number of x's and y's
(ignoring the signs)
e.g. 2x & 2x or -3y & +3y

EXAMPLE: Solve

$$\begin{aligned} 6x - y &= -4 \\ 3x - 2y &= 1 \end{aligned}$$

$$\begin{aligned} 12x - 2y &= -8 \\ 3x - 2y &= 1 \end{aligned}$$

SUBTRACT,

$$\div 9 \left(\begin{array}{l} 9x = -9 \\ x = -1 \end{array} \right) \div 9$$

$$\begin{aligned} 3x - 2y &= 1 \\ 3 \times -1 - 2y &= 1 \\ -3 - 2y &= 1 \\ -2y &= 4 \\ y &= -2 \end{aligned}$$

*Multiply one
equation by 2 and
the other by 5 to
get 10b & -10b
(We could also
have made 14a
and 14a)*

*Double the first
equation to get
-2y and -2y
(We could also
have doubled the
second equation
to get 6x and 6x)*

*Now solve as
normal*

scaling both...

EXAMPLE: Solve

$$\begin{aligned} 2a + 2b &= 16 \\ 7a - 5b &= 20 \end{aligned}$$

$$\begin{aligned} 10a + 10b &= 80 \\ 14a - 10b &= 40 \end{aligned}$$

ADD,

$$\div 24 \left(\begin{array}{l} 24a = 120 \\ a = 5 \end{array} \right) \div 24$$

$$\begin{aligned} 10a + 10b &= 80 \\ 10 \times 5 + 10b &= 80 \\ 50 + 10b &= 80 \\ 10b &= 30 \\ b &= 3 \end{aligned}$$

*We could substitute a=5 and b=3
into both original equations, to
check the solution*

problem-solving...

EXAMPLE:

4 nuts and 2 bolts weigh 20 grams.
3 nuts and 4 bolts weigh 29 grams.
Find the weight of a nut and the weight of a bolt.

$$\begin{aligned}4n + 2b &= 20 \\ 3n + 4b &= 29\end{aligned}$$

Write
simultaneous
equations to
represent the
problem

$$\begin{aligned}12n + 6b &= 60 \\ 12n + 16b &= 116\end{aligned}$$

Scale up if
needed

SUBTRACT,

$$\begin{array}{r} \div 24 \quad \left(\begin{array}{l} -10b = -56 \\ b = 5.6 \end{array} \right) \div 24 \end{array}$$

$$4n + 2b = 20$$

$$4n + 2 \times 5.6 = 20$$

$$4n + 11.2 = 20$$

$$4n = 8.8$$

$$n = 2.2$$

A nut is 2.2g, a bolt is 5.6g

Solve as normal,
answering the question
in context