chaseterraceacademy

Year 10 Maths

Unit 6: Solids

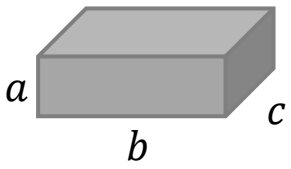
MATHOPEDIA

Volume is a measurement of the space taken up by a 3D solid.
We measure volume in units like cm^3 or m^3.

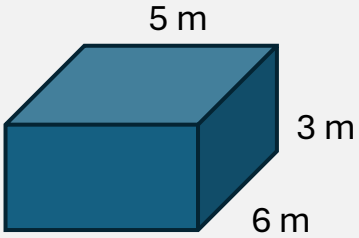
volume of a cuboid...

Volume of cuboid:

$a \times b \times c$



EXAMPLE:
Calculate the volume of the cuboid.



Multiply the three ***dimensions***

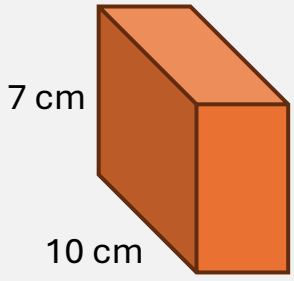
5 x 3 x 6

= 90 m^3

Include the units with the answer

finding sides...

EXAMPLE: The cuboid has volume 210 cm^3.
Find its width (w).



Form an equation using the volume

7 x 10 x w = 210

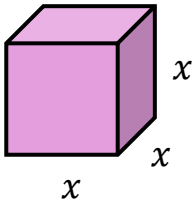
And solve it

70 x w = 210

w = 3 cm

EXAMPLE: A cube has volume 343 m^3.
Find its side length.

All three sides of a cube are the same length



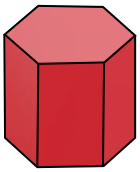
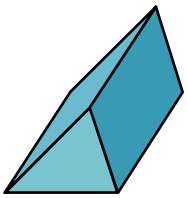
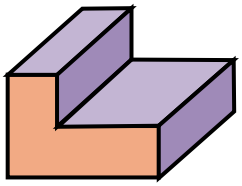
The opposite of cubing is a ***cube root***

$x^3 = 343$

$x = \sqrt[3]{343}$

$= 7$ metres

A **prism** is a 3D shape which has all flat faces and the same shape 'running through the middle' – the **cross-section**.

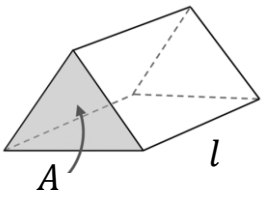




Note that a cylinder <u>isn't</u> a prism, because it doesn't have all flat faces.

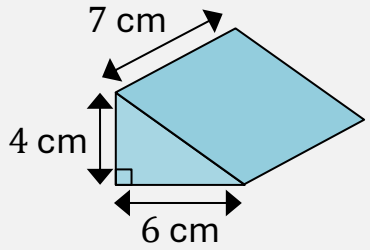
volume of a prism...

Volume of prism:

Area of cross-section x length



EXAMPLE: Calculate the volume of the prism.



Calculate the area of the cross-section (triangle)

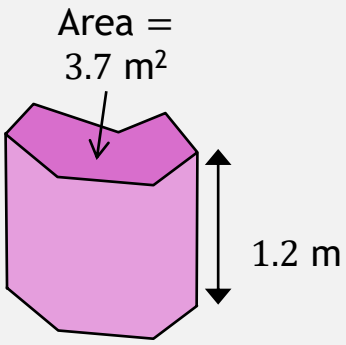
$$\frac{1}{2} \times 4 \times 6 = 12$$

Area x length

$$12 \times 7$$

$$= 94 \text{ cm}^3$$

EXAMPLE: Calculate the volume of the prism.

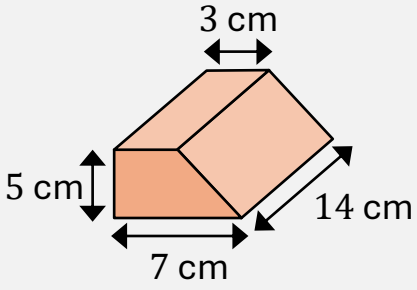


Area x length

3.7 x 1.2

$= 4.44 \text{ cm}^3$

EXAMPLE: Calculate the volume of the prism.



Calculate the area of the cross-section (trapezium)

$$\frac{1}{2}(7 + 3) \times 5 = 25$$

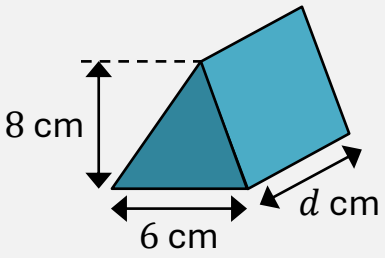
Area x length

25 x 14

$= 350 \text{ cm}^3$

sides of prisms...

EXAMPLE: The volume of the prism is 180 cm³. Find the value of d.



Form an equation for area x length

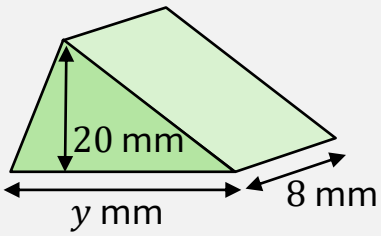
$$\frac{6 \times 8}{2} \times d = 180$$

Solve it

$$24d = 180$$

$$d = 7.5$$

EXAMPLE:
The volume of the prism is 3640 mm³.
Find the value of y.



Form an equation for area x length

$$\frac{20 \times y}{2} \times 8 = 180$$

Solve it

$$\frac{20y}{2} = 22.5$$

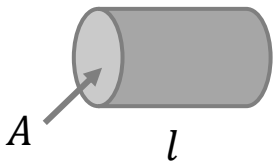
$$20y = 45$$

$$y = 2.25$$

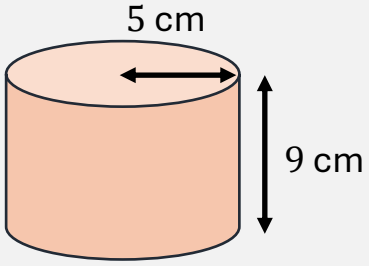
volume of a cylinder...

Volume of cylinder:

$$area\ of\ circle \times length$$



EXAMPLE:
Calculate the volume.
(a) Give your answer in terms of π.



Calculate the area of the cross-section (circle)

$$A = \pi r^2$$
$$= \pi \times 5^2$$
$$= 25\,\pi$$

Multiply by the length (height in this case)

$$V = 25\pi \times 9$$

Leave the answer in terms of π

$$= 225\,\pi \text{ cm}^3$$

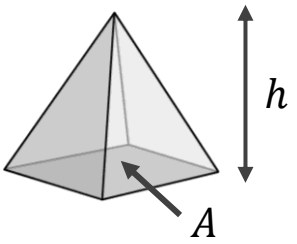
(b) Give your answer correct to 2 decimal places.

$$225 \times \pi$$
$$= 706.8583471$$

Use a calculator then round the answer

$$= 706.86 \text{ cm}^3$$

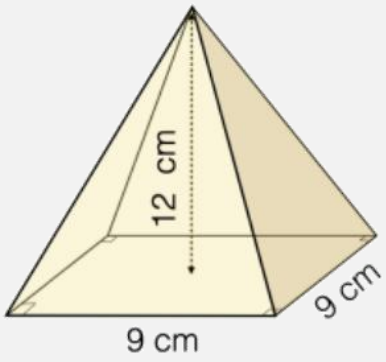
volume of a pyramid...



Volume of pyramid:

$$\frac{1}{3} \times \begin{matrix} area\ of \\ base \end{matrix} \times height$$

EXAMPLE: Calculate the volume of the square-based pyramid.



Calculate the area of the (square) base

$$A = 9 \times 9 = 81$$

Use the formula to calculate the volume

$$V = \frac{1}{3} \times 81 \times 12$$

$$= 324 \text{ cm}^3$$

other solids...

Formulas for **spheres** or **cones** will be given to you in the question.

EXAMPLE: Calculate the volume of a sphere with radius 6 metres. Give your answer in terms of π.

The formula for the volume of a sphere is

$$V = \frac{4}{3}\pi r^3$$

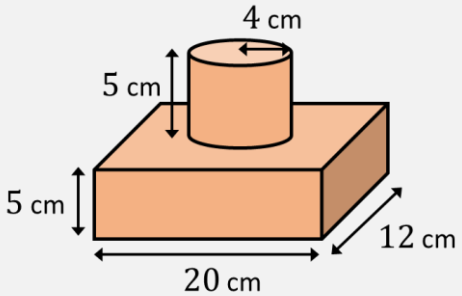
Use the formula given in the question

$$V = \frac{4}{3} \times \pi \times 6^3$$

$$= 288\ \pi \quad \text{m}^3$$

composite solids...

EXAMPLE: Calculate the volume to 1 decimal place.



Cylinder,

$$A = \pi \times 4^2$$
$$= 16\pi$$

$$V = 16\pi \times 5$$
$$= 80\pi$$

Cuboid,

$$V = 5 \times 20 \times 12$$
$$= 1200$$

Work out the volume of each part separately, then combine

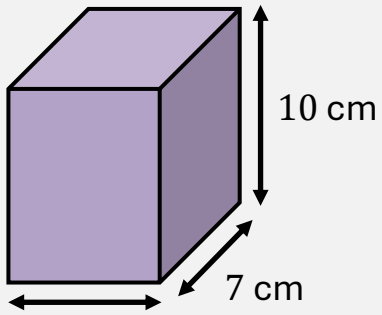
Total volume,

$$V = 1200 + 80\pi$$
$$= 1451.327412$$
$$= 1451.3 \text{ cm}^3 \text{ (1d.p.)}$$

The **surface area** of a solid is the total of the areas of all its faces, added together.

cubes and cuboids...

EXAMPLE: Calculate the surface area.



Calculate the area of each different face

Front: $A = 6 \times 10 = 60$

Top: $A = 6 \times 7 = 42$

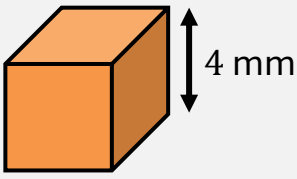
Side: $A = 7 \times 10 = 70$

Combine: the cuboid has 2 of each face, 6 faces altogether

$2 \times 60 + 2 \times 42 + 2 \times 70$

$= 344 \text{ cm}^2$

EXAMPLE: Calculate the surface area of the cube.



$A = 4 \times 4 = 16$

$V = 16 \times 6$

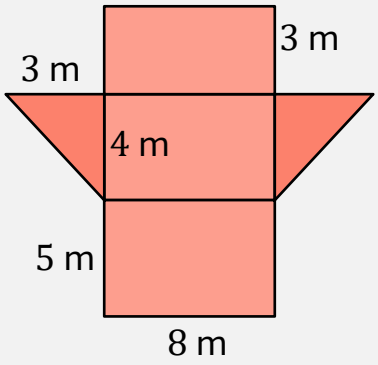
$= 96 \text{ mm}^2$

Calculate the area of one face

A cube has 6 identical faces

triangular prisms...

EXAMPLE: Here is the net of a triangular prism. Find the surface area.



Calculate the area of each face

Triangles (x2),

$\frac{1}{2} \times 3 \times 4 = 6$

Rectangles,

$5 \times 8 = 40$

$4 \times 8 = 32$

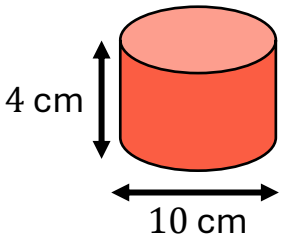
$3 \times 8 = 24$

Combine all the faces

$2 \times 6 + 40 + 32 + 24$

$= 108 \text{ m}^2$

cylinders...



EXAMPLE: Calculate the **exact** surface area.

Circle,

$= \pi \times 5^2 = 25\pi$

The curved surface 'flattens' into a rectangle. The top edge is the circumference of the circle.

Rectangle,

$= C \times 4$

$= (\pi \times 10) \times 4$

$= 40\pi$

Total,

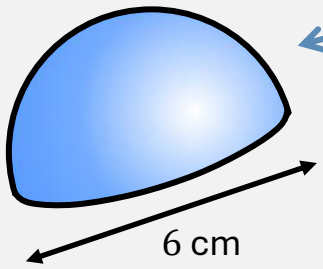
$40\pi + 25\pi$

$= 65\pi \text{ cm}^2$

*An **exact** answer, in terms of π*

using formulas...

EXAMPLE:
Calculate the surface area of the hemisphere.
Give your answer in terms of π.
(The formula for the surface area of a sphere is $A = 4\pi r^2$)



A hemisphere is half a sphere. It has a circular face and a curved surface.

Area of circular face,

$$A = \pi r^2$$
$$= \pi \times 3^2$$
$$= 9\,\pi$$

Area of the circular face (radius = 3cm)

Area of curved surface,

$$A = \frac{1}{2} \times 4\pi r^2$$
$$= 2 \times \pi \times 3^2$$
$$= 18\,\pi$$

Half the area for half a sphere

Total surface area,

$$18\,\pi + 9\,\pi$$
$$= 27\,\pi \text{ cm}^2$$

Leave in terms of π

practical problems...

'Melting down' is a classic volume problem

EXAMPLE:
10 solid metal cubes, each of side 4cm, are melted down.

The metal is reformed into a cuboid measuring 12cm by 10cm by h cm.

Calculate the value of h.

Each cube,

$$V = 4 \times 4 \times 4$$
$$= 64$$

This is the total amount of metal available

10 cubes,

$$V = 64 \times 10$$
$$= 640 \text{ (cm}^3\text{)}$$

cuboid,

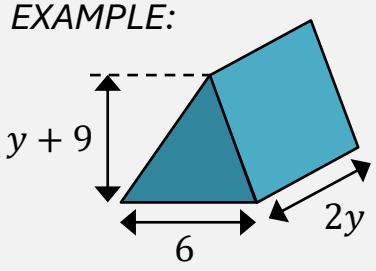
$$12 \times 10 \times h = 640$$
$$120 \times h = 640$$

The cuboid uses the same 640 cm^3 of metal

$$h = 640 \div 120$$
$$= \frac{16}{3} \quad (\text{or } 5.\dot{3})$$

algebraic problems...

EXAMPLE:



Find an expression for
(a) the volume

$$A = \frac{6(y+9)}{2}$$
$$= 3(y+9)$$

Calculate the area of the cross-section (triangle)

$$V = 3(y+9) \times 2y$$
$$= 6y(y+9)$$

Multiply the area by the length

(b) the surface area

Triangle,

$$\frac{1}{2} \times 6 \times (y+9)$$
$$= 3(y+9)$$
$$= 3y + 27$$

Rectangles,

$$6(y+9) = 6y + 54$$
$$2y(y+9) = 2y^2 + 9y$$
$$2y \times 6 = 12y$$

Find the areas of the faces

Total surface area,

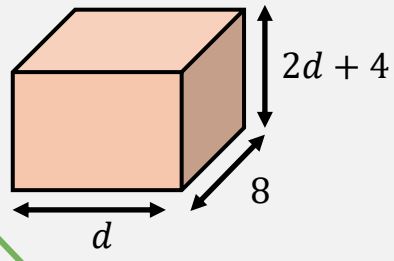
$$3y + 27 + 3y + 27 + 2y^2 + 9y + 12y$$
$$= 2y^2 + 27y + 54$$

Add together the faces: 2 triangles & 3 rectangles

advanced algebra...

EXAMPLE:

The surface area of the cuboid is 352 cm^2.
Find the value of d.



Find the surface area of each face

Top:

$$A = 8 \times d = 8d$$

Side:

$$A = 8(2d+4)$$
$$= 16d + 32$$

Front:

$$A = d(2d+4)$$
$$= 2d^2 + 4d$$

Write an expression for the total surface area (two of each face)

surface area,

$$2(8d + 16d + 32 + 2d^2 + 4d)$$
$$= 2(2d^2 + 28d + 32)$$
$$= 4d^2 + 56d + 64$$

Create an equation

using 352,

$$4d^2 + 56d + 64 = 352$$
$$4d^2 + 56d - 288 = 0$$
$$d^2 + 14d - 72 = 0$$

Solve by factorising (or quadratic formula)

$$(d+18)(d-4) = 0$$
$$d + 18 = 0 \text{ or } d - 4 = 0$$
$$d = -18 \text{ or } d = 4$$
$$d = 4$$

d is a length, so can't be -18 cm