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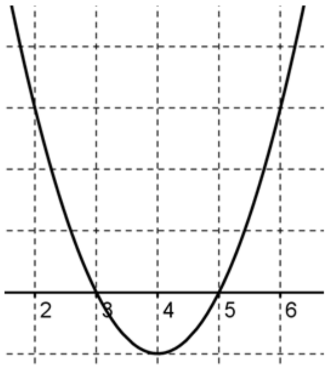
Year 10 Maths

Unit 5: Equations

MATHOPEDIA



In a **quadratic** equation, the highest power of x is 2.

The graph of a quadratic function makes this U-shaped curve, called a **parabola**.

The **roots** of a quadratic function are the values of x where the graph meets the x-axis. We can identify these points by solving a quadratic equation.

For the graph on the left, the roots are 3 and 5.

Factorising x^2...

EXAMPLE:
Factorise $x^2 + 7x + 12$

+ ×

1, 12
3, 4
2, 6

List all the pairs that multiply to make 12. Find the pair that also adds to 7

$= (x + 3)(x + 4)$

The brackets need x & x to make x^2. They also have our chosen pair of numbers: 3, 4

EXAMPLE:
Factorise $x^2 - 3x - 10$

+ ×

5, -2
-5, 2
10, -1
-10, 1

$= (x - 5)(x + 2)$

factorise & solve...

EXAMPLE:
Solve by factorising:
$x^2 - 7x + 10 = 0$

+ ×

First factorise. (We need two negative numbers to multiply to make +10)

-1, -10
-2, -5

$(x - 2)(x - 5) = 0$

$x - 2 = 0$ or $x - 5 = 0$

$x = 2$ or $x = 5$

The solutions are the values of x which make the brackets 0.

Hence identify the roots of the function:
$y = x^2 - 7x + 10$

$x = 2$ and $x = 5$

The solutions are the roots: where the graph $y = x^2 - 7x + 10$ would cross the x-axis.

quadratic formula...

We can solve *any* quadratic equation using the **quadratic formula,** including quadratics that can't be factorised.

In general, for the equation: $ax^2 + bx + c = 0$

the solutions are given by: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXAMPLE:
Solve $3x^2 - 2x - 7 = 0$
Give solutions correct to 3 significant figures.

Identify the values of a, b & c

$$3x^2 - 2x - 7 = 0$$

$a = 3$ $b = -2$ $c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute into the quadratic formula. Use brackets on the calculator for negative values.

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 3 \times (-7)}}{2 \times 3}$$

$= 1.896805253$
or -1.230138587

Replace the '±' with '+' then '–' to get two solutions. Write these in full.

$= 1.90$ or -1.23

Round the solutions as we were asked to in the question

Solutions can be checked by substituting them into the original equation

$x + y = 10$ This equation has an infinite number of solutions (e.g. $x = 3.5$ and $y = 6.5$).

$x + y = 10$
$2x - y = 14$

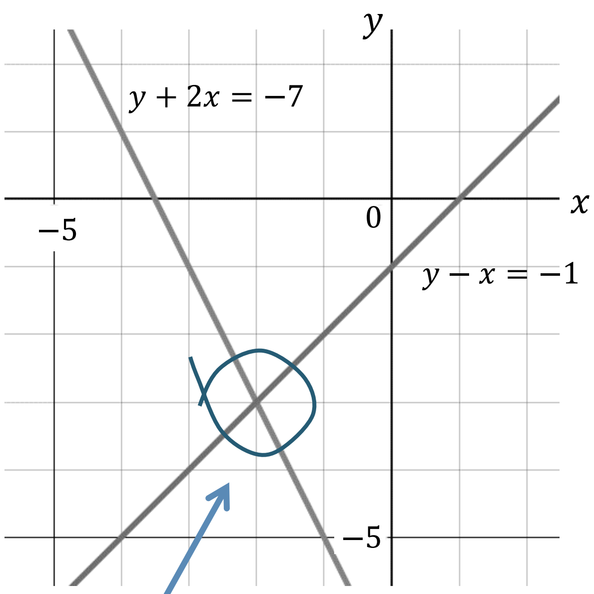
However, when we combine it with another equation, there is only one pair of values that are solutions to both equations. ($x = 8$ and $y = 2$)

We call these pairs of equations, **simultaneous equations**.

solving graphically...

EXAMPLE:
Use the graph to estimate the solutions to the simultaneous equations:

$y + 2x = -7$
$y - x = -1$



$x = -2, \quad y = -3$

Notice that the equations match the lines on the graph

The solutions are the point where the two graphs intersect

solve by adding...

EXAMPLE: Solve
$x + y = 10$
$2x - y = 14$

ADD,

$3x = 24$ (÷ 3)
$x = 8$

$x + y = 10$
$8 + y = 10$
$y = 2$

Adding the two equations eliminates y:
$x + 2x = 3x$
$y + -y = 0$
$10 + 14 = 28$

Subtracting eliminates x
$3y - -y = 4y$

Substitute to find the other value

or by subtracting...

EXAMPLE: Solve
$2x + 3y = 25$
$2x - y = 13$

SUBTRACT,

$4y = 12$ (÷ 4)
$y = 3$

$2x - y = 13$
$2x - 3 = 13$
$x = 8$

add or subtract?

We can decide whether to add or subtract using the 'STOP' rule.

SAME – **T**AKE
OPPOSITE – **P**LUS

$$2x + 3y = 8$$
$$2x - 4y = -6$$

2x and 2x are both positive.
We would subtract these equations.
(SAME – TAKE)

$$3x - 6y = 2$$
$$5x - 6y = 7$$

-6y and -6y are both negative.
We would subtract these equations.
(SAME – TAKE)

$$5x + 2y = 11$$
$$7x - 2y = 1$$

+2y and +2y have opposite signs.
We would add these equations.
(OPPOSITE – PLUS)

scaling up...

To eliminate a variable, we need the same number of x's and y's (ignoring the signs)
e.g. 2x & 2x or -3y & +3y

EXAMPLE: Solve
$$6x - y = -4$$
$$3x - 2y = 1$$

Double the first equation to get -2y and -2y (We could also have doubled the second equation to get 6x and 6x)

$$\mathbf{12x - 2y = -8}$$
$$3x - 2y = 1$$

SUBTRACT,

$$9x = -9$$
$$\div 9 \qquad x = -1 \qquad \div 9$$

Now solve as normal

$$3x - 2y = 1$$
$$3 \times -1 - 2y = 1$$
$$-3 - 2y = 1$$
$$-2y = 4$$
$$y = -2$$

scaling both...

EXAMPLE: Solve
$$2a + 2b = 16$$
$$7a - 5b = 20$$

Multiply one equation by 2 and the other by 5 to get 10b & -10b (We could also have made 14a and 14a)

$$10a + 10b = 80$$
$$14a - 10b = 40$$

ADD,

$$24a = 120$$
$$\div 24 \qquad a = 5 \qquad \div 24$$

$$10a + 10b = 80$$
$$10 \times 5 + 10b = 80$$
$$50 + 10b = 80$$
$$10b = 30$$
$$b = 3$$

We could substitute a=5 and b=3 into both original equations, to check the solution

problem-solving...

EXAMPLE:
4 nuts and 2 bolts weigh 20 grams.
3 nuts and 4 bolts weigh 29 grams.
Find the weight of a nut and the weight of a bolt.

$$4n + 2b = 20$$
$$3n + 4b = 29$$

Write simultaneous equations to represent the problem

$$12n + 6b = 60$$
$$12n + 16b = 116$$

Scale up if needed

SUBTRACT,

$$-10b = -56$$
$$b = 5.6$$

÷ 24 ÷ 24

$$4n + 2b = 20$$
$$4n + 2 \times 5.6 = 20$$
$$4n + 11.2 = 20$$
$$4n = 8.8$$
$$n = 2.2$$

A nut is 2.2g, a bolt is 5.6g

Solve as normal, answering the question in context