

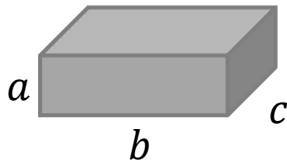


**Volume** is a measurement of the space taken up by a 3D solid.  
We measure volume in units like  $\text{cm}^3$  or  $\text{m}^3$ .

### volume of a cuboid...

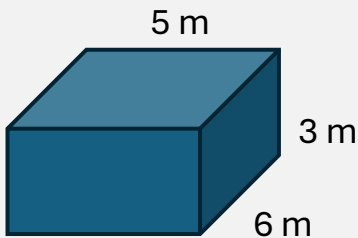
**Volume of cuboid:**

$$a \times b \times c$$



**EXAMPLE:**

Calculate the volume of the cuboid.



$$5 \times 3 \times 6$$

$$= 90 \text{ m}^3$$

Include the units with the answer

Multiply the three dimensions

Form an equation using the volume

And solve it

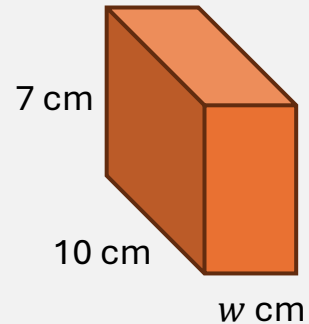
$$7 \times 10 \times w = 210$$

$$70 \times w = 210$$

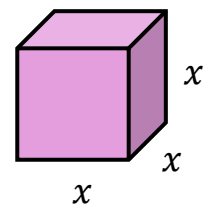
$$w = 3 \text{ cm}$$

### finding sides...

**EXAMPLE:** The cuboid has volume  $210 \text{ cm}^3$ . Find its width ( $w$ ).



**EXAMPLE:** A cube has volume  $343 \text{ m}^3$ . Find its side length.



$$x^3 = 343$$

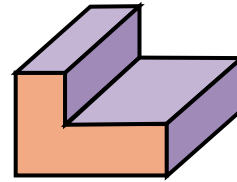
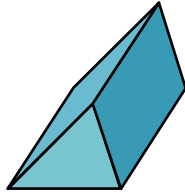
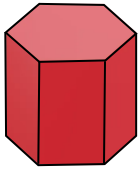
$$x = \sqrt[3]{343}$$

$$= 7 \text{ metres}$$

All three sides of a cube are the same length

The opposite of cubing is a cube root

A **prism** is a 3D shape which has all flat faces and the same shape 'running through the middle' – the **cross-section**.

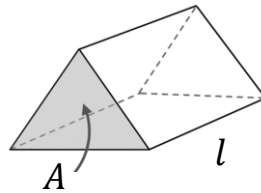


Note that a cylinder isn't a prism, because it doesn't have all flat faces.

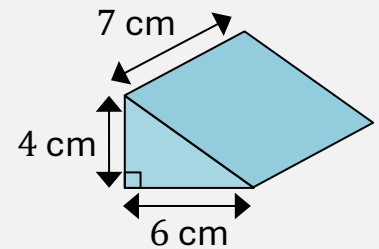
volume of a prism...

**Volume of prism:**

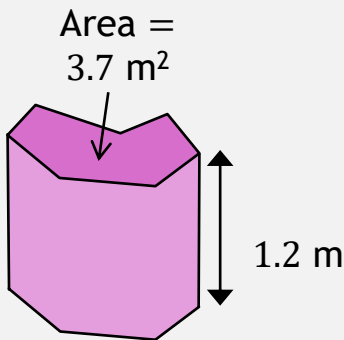
Area of cross-section  $\times$  length



**EXAMPLE:** Calculate the volume of the prism.



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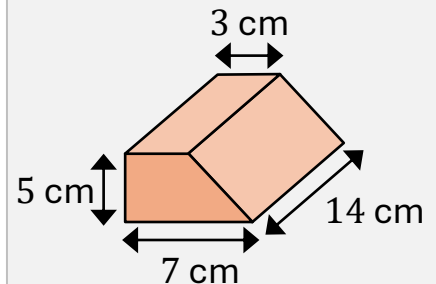
Calculate the area of the cross-section (triangle)

Area  $\times$  length

$$\frac{1}{2} \times 4 \times 6 = 12$$

$$12 \times 7 = 84 \text{ cm}^3$$

**EXAMPLE:** Calculate the volume of the prism.



Area  $\times$  length

$$3.7 \times 1.2 = 4.44 \text{ cm}^3$$

Calculate the area of the cross-section (trapezium)

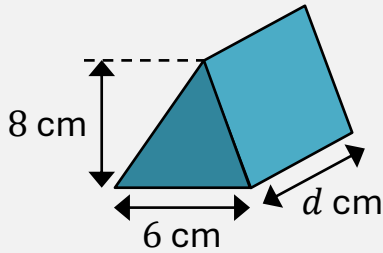
Area  $\times$  length

$$\frac{1}{2} (7 + 3) \times 5 = 25$$

$$25 \times 14 = 350 \text{ cm}^3$$

## sides of prisms...

**EXAMPLE:** The volume of the prism is  $180 \text{ cm}^3$ . Find the value of  $d$ .



$$\frac{6 \times 8}{2} \times d = 180$$

$$24d = 180$$

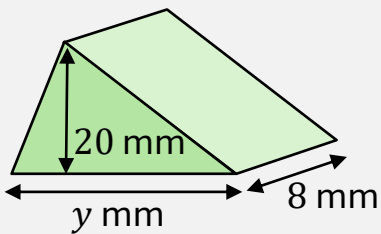
$$d = 7.5$$

Form an equation for area  $\times$  length

Solve it

**EXAMPLE:**

The volume of the prism is  $3640 \text{ mm}^3$ . Find the value of  $y$ .



$$\frac{20 \times y}{2} \times 8 = 180$$

$$\frac{20y}{2} = 22.5$$

$$20y = 45$$

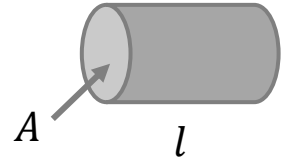
$$y = 2.25$$

Form an equation for area  $\times$  length

Solve it

**Volume of cylinder:**

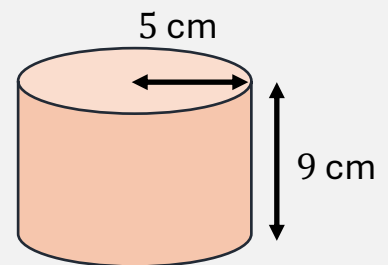
area of circle  $\times$  length



**EXAMPLE:**

Calculate the volume.

(a) Give your answer in terms of  $\pi$ .



Calculate the area of the cross-section (circle)

Multiply by the length (height in this case)

Leave the answer in terms of  $\pi$

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 5^2 \\ &= 25\pi \end{aligned}$$

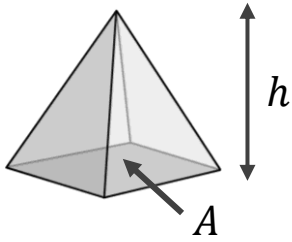
$$\begin{aligned} V &= 25\pi \times 9 \\ &= 225\pi \text{ cm}^3 \end{aligned}$$

(b) Give your answer correct to 2 decimal places.

$$\begin{aligned} &225 \times \pi \\ &= 706.8583471 \\ &= 706.86 \text{ cm}^3 \end{aligned}$$

Use a calculator, then round the answer

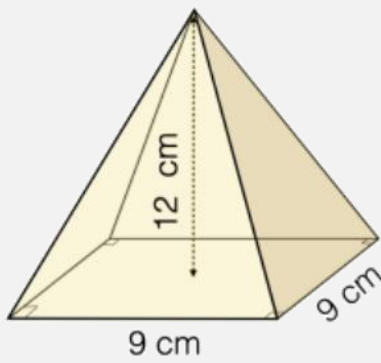
## volume of a pyramid...



**Volume of pyramid:**

$$\frac{1}{3} \times \text{area of base} \times \text{height}$$

**EXAMPLE:** Calculate the volume of the square-based pyramid.



$$A = 9 \times 9 = 81$$

$$V = \frac{1}{3} \times 81 \times 12$$

$$= 324 \text{ cm}^3$$

Calculate the area of the (square) base

Use the formula to calculate the volume

Use the formula given in the question

## other solids...

Formulas for **spheres** or **cones** will be given to you in the question.

**EXAMPLE:** Calculate the volume of a sphere with radius 6 metres. Give your answer in terms of  $\pi$ .

The formula for the volume of a sphere is

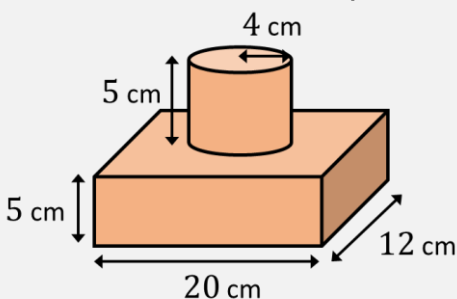
$$V = \frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} \times \pi \times 6^3$$

$$= 288 \pi \text{ m}^3$$

## composite solids...

**EXAMPLE:** Calculate the volume to 1 decimal place.



Cylinder,

$$A = \pi \times 4^2$$

$$= 16\pi$$

$$V = 16\pi \times 5$$

$$= 80\pi$$

Work out the volume of each part separately, then combine

Cuboid,

$$V = 5 \times 20 \times 12$$

$$= 1200$$

Total volume,

$$V = 1200 + 80\pi$$

$$= 1451.327412$$

$$= 1451.3 \text{ cm}^3 \text{ (1d.p.)}$$

## practical problems...

### EXAMPLE:

10 solid metal cubes, each of side 4cm, are melted down.

The metal is reformed into a cuboid measuring 12cm by 10cm by  $h$  cm.

Calculate the value of  $h$ .

Each cube,

$$V = 4 \times 4 \times 4$$

10 cubes,

$$\begin{aligned} V &= 64 \times 10 \\ &= 640 \text{ (cm}^3\text{)} \end{aligned}$$

cuboid,

$$\begin{aligned} 12 \times 10 \times h &= 640 \\ 120 \times h &= 640 \end{aligned}$$

$$h = 640 \div 120$$

$$= \frac{16}{3} \text{ (or } 5.\dot{3}\text{)}$$

*'Melting down' is a classic volume problem*

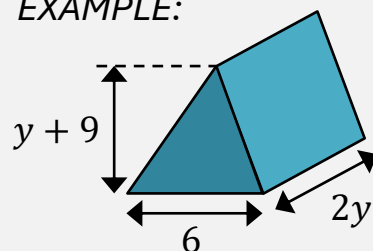
*Calculate the area of the cross-section (triangle)*

*This is the total amount of metal available*

*The cuboid uses the same 640 cm<sup>3</sup> of metal*

## using algebra...

### EXAMPLE:



Find an expression for the volume of the prism.

$$A = \frac{6(y + 9)}{2}$$

$$= 3(y + 9)$$

$$\begin{aligned} V &= 3(y + 9) \times 2y \\ &= 6y(y + 9) \end{aligned}$$

*Multiply the area by the length*