Year 10 Maths

Unit 7: Fractions

MATHOPEDIA

In the UK, **recurring** decimals are represented by a dot above the digit(s).
e.g.

$0.3\dot{4} = 0.34444\ldots$
$0.3\dot{1}\dot{4} = 0.3141414\ldots$
$0.\dot{3}1\dot{4} = 0.314314\ldots$

fraction to decimal...

EXAMPLE:
Convert $\frac{4}{9}$ to a decimal

$$9\overline{)4.^{4}0^{4}0^{4}0}$$

$= 0.\dot{4}$

Use a 'bus stop' to divide 4 by 9

Write the answer using recurring 'dot' notation

decimal to fraction...

EXAMPLE: Convert $0.3\dot{4}$ to a fraction

Let $x = 0.3\dot{4} = 0.344\ldots$

$10x = 3.444\ldots$

$$\begin{array}{r} 3.444\ldots \\ -\ 0.344\ldots \\ \hline 3.100\ldots \end{array}$$

$9x = 3.1$

$x = \frac{3.1}{9} = \frac{31}{90}$

Multiplying by 10 'lines up' all the 4's

10x – x = 9x

Divide both sides by 9 then simplify

other decimal types...

EXAMPLE: Convert $0.3\dot{1}\dot{4}$ to a fraction

Let $x = 0.3\dot{1}\dot{4}$
$= 0.3141414\ldots$

$100x = 31.4141414\ldots$

*This time, multiplying by **100** lines up the 4's and 1's*

$$\begin{array}{r} 31.41414\ldots \\ -\ 0.31414\ldots \\ \hline 31.10000\ldots \end{array}$$

100x – x = 99x

$99x = 31.1$

$x = \frac{31.1}{99} = \frac{311}{990}$

EXAMPLE: Convert $0.\dot{3}1\dot{4}$ to a fraction

Let $x = 0.\dot{3}1\dot{4}$
$= 0.314314\ldots$

$1000x = 314.314314\ldots$

*This time, multiplying by **1000** lines up the '314's*

$$\begin{array}{r} 314.314314\ldots \\ -\ 0.314314\ldots \\ \hline 314.000000\ldots \end{array}$$

1000x – x = 999x

$999x = 314$

$x = \frac{314}{999}$

ordering fractions...

We use a common denominator to put fractions in size order.

EXAMPLE:
Write these fractions in order of size, starting with the smallest:

$$\frac{4}{15} \quad \frac{2}{5} \quad \frac{3}{10}$$

15, 5 and 10 all go into a common multiple of 30

$$\frac{4}{15} = \frac{8}{30}$$
$$\frac{2}{5} = \frac{12}{30}$$
$$\frac{3}{10} = \frac{9}{30}$$

Convert all fractions to the common denominator

$$\frac{8}{30} \quad \frac{9}{30} \quad \frac{12}{30}$$

Put the fractions in size order

$$\frac{4}{15} \quad \frac{3}{10} \quad \frac{2}{5}$$

Write the original fractions in the correct order

ordering FDP...

We can order fractions, decimals & percentages by converting them all to decimals.

EXAMPLE: Write in ascending order:

$$\frac{17}{20} \quad 0.871 \quad \frac{5}{6} \quad 87\%$$

Convert to decimals (with or without a calculator)

17 ÷ 20 = 0.85
5 ÷ 6 = 0.833...
87% = 0.87

0.8333, 0.85, 0.87, 0.871

Order the decimals and convert back

$$\frac{5}{6} \quad \frac{17}{20} \quad 87\% \quad 0.871$$

We calculate with algebraic fractions using the same rules we use for calculating with numerical fractions.

multiply / divide...

EXAMPLE: Simplify $\frac{1}{x} \times \frac{x^2}{2}$

$$\frac{1}{x} \times \frac{x^2}{2} = \frac{x^2}{2x}$$

Multiply the numerators and denominators

$$= \frac{x}{2}$$

Simplify, dividing the numerator & denominator by x.

EXAMPLE: Simplify $\frac{x^2 - 4x}{(x+1)(x-1)} \div \frac{x+4}{x+1}$

$$= \frac{x^2 - 4x}{(x+1)(x-1)} \times \frac{x+1}{x+4}$$

'Keep-flip-change' (multiply by the reciprocal)

$$= \frac{(x^2 - 4x)(x+1)}{(x+1)(x-1)(x-4)}$$

Multiply the numerators and denominators

$$= \frac{x^2 - 4x}{(x-1)(x-4)}$$

Divide the numerator and denominator by x + 1

$$= \frac{x(x-4)}{(x-1)(x-4)}$$

By factorising we can simplify further, dividing by x – 4

$$= \frac{x}{(x-1)}$$

add / subtract...

EXAMPLE: Simplify $\frac{x+1}{2} - \frac{x-1}{3}$

Use a common denominator, multiplying the numerators

$$= \frac{3(x+1)}{6} - \frac{2(x-1)}{6}$$

Notice that the '+2' term ends up positive

$$= \frac{3x + 3 - (2x - 2)}{6}$$

$$= \frac{3x + 3 - 2x + 2}{6}$$

$$= \frac{x+5}{6}$$

EXAMPLE: Simplify $\frac{1}{x} + \frac{2}{x^2}$

The lowest common denominator is x^2

$$= \frac{x}{x^2} + \frac{2}{x^2}$$

$$= \frac{x+2}{x^2}$$

EXAMPLE: Simplify $\frac{3}{x-1} - \frac{4}{x}$

$$= \frac{3x}{x(x-1)} - \frac{4(x-1)}{x(x-1)}$$

$$= \frac{3x - (4x - 4)}{x(x-1)}$$

$$= \frac{-x+4}{x(x-1)}$$

Notice the '+4' term is positive

fractional change...

EXAMPLE:

Increase £450 by $\frac{3}{5}$

$\frac{5}{5} + \frac{3}{5} = \frac{8}{5}$

$\frac{8}{5}$ of 450

450 ÷ 5 = 90

90 x 8 = £720

EXAMPLE:

Decrease £450 by $\frac{3}{5}$

$\frac{5}{5} - \frac{3}{5} = \frac{2}{5}$

$\frac{2}{5}$ of 450

450 ÷ 5 = 90

90 x 2 = £180

repeated change...

EXAMPLE: On Monday, Mike runs 20 km. Each day, he reduces the length of his run by $\frac{1}{4}$ How far does he run on Wednesday?

$\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$

$20 \times \left(\frac{3}{4}\right)^2$

$= 20 \times \frac{9}{16}$

$= 20 \div 16 \times 9$

$= 11.25$ km

Reduces twice by Wednesday