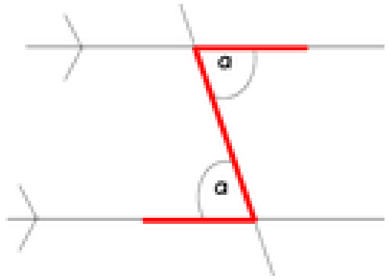
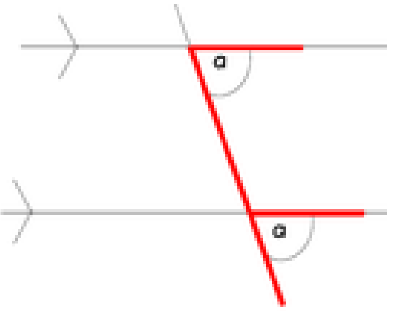
parallel lines...

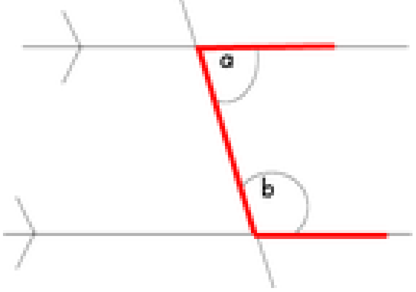
Alternate angles are equal.
(Inside the parallel lines – look for the 'Z' shape)



Corresponding angles are equal.
(Any matching pair of angles, e.g. both bottom right ones)

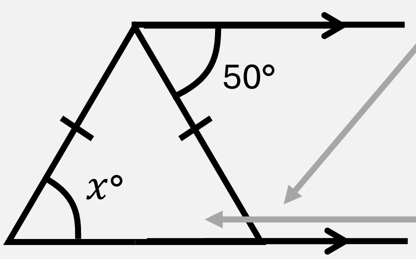


Co-interior angles add up to 180°.
(Look for the 'C' or 'U' shape)



EXAMPLE:

Find the size of angle x.
Give **reasons** to show how you decide.



"Give reasons" means write down the angle rules used.

180 – 50 = 130
Co-interior angles add up to 180°

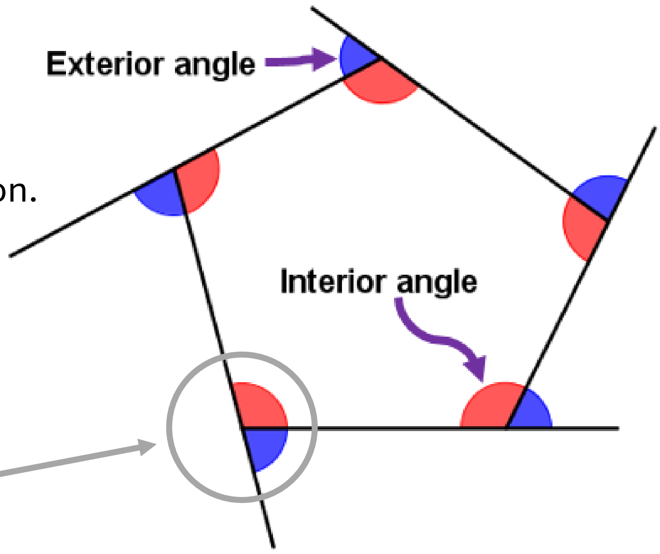
180 – 130 = 50
Angles on a straight line add up to 180°

x = 50
Base angles in an isosceles triangle are equal.

exterior angles...

A **polygon** is a shape with straight sides.
A **regular** polygon has all equal sides, and all equal angles.

The diagram shows the **interior** and **exterior** angles of a polygon.



The exterior angles of any polygon always add up to 360°

Each pair of an interior and an exterior angle adds up to 180°, because they are angles on a straight line.

EXAMPLE: Calculate the size of the exterior angle in a regular 12-sided polygon.

360 ÷ 12 = 30°

Exterior angles add up to 360°, divided by the number of sides/angles

EXAMPLE: The size of each interior angle in a regular polygon is 150°. Work out how many sides the polygon has.

180 – 150 = 30

Each exterior angle is 30° as a pair of an interior and exterior angle add up to 180°

360 ÷ 30 = 12 sides

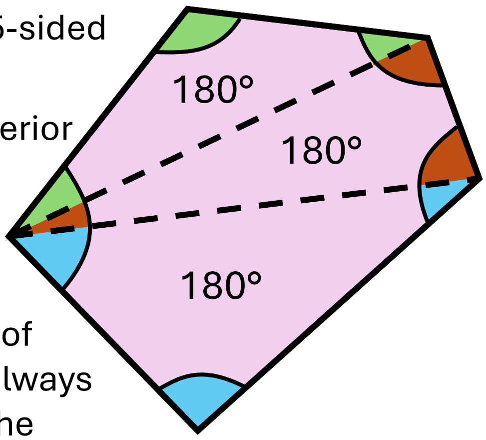
Exterior angles add up to 360, so there are 12 angles (and 12 sides)

interior angles...

The sum (total) of the interior angles can be found by dividing the polygon into triangles. Each triangle has 180°.

e.g. For this 5-sided polygon,

Sum of interior angles
= 3 x 180
= 540°



The number of triangles is always 2 less than the number of sides.

<u>Formula:</u>

The sum of the interior angles of an n-sided polygon is

$$(n-2) \times 180$$

EXAMPLE: Find the sum of the interior angles of a decagon (10 sides)

$$(n-2) \times 180$$
$$(10-2) \times 180$$
$$= 1440°$$

Substitute the number of sides into the formula

EXAMPLE:
Work out the size of each interior angle in a regular octagon

$$(n-2) \times 180$$
$$(8-2) \times 180$$
$$= 1080°$$

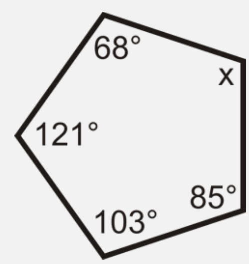
Find the sum of interior angles in an octagon

$$1080 \div 8 = 135°$$

As it's regular, all 8 interior angles will be the same size

missing angles...

EXAMPLE: Calculate the size of angle x.



Find the sum of the interior angles in a pentagon

(5 – 2) x 180 = 540

68 + 121 + 103 + 85
= 377

x = 540 – 377 = 163°

Add up the other angles, then subtract from the expected total

advanced polygons...

EXAMPLE: An interior angle of a regular polygon is 11 times its exterior angle.
Work out the number of sides of the polygon.

$$\text{exterior angle} = \frac{360}{n}$$

Find expressions for the interior and exterior angles, with n as the unknown number of sides

$$\text{interior angle} = \frac{(n-2)\times 180}{n}$$

Rewrite the statement in the question as an equation

$$\frac{(n-2)\times 180}{n} = 11 \times \frac{360}{n}$$

$$\frac{(n-2)\times 180}{n} = \frac{3960}{n}$$

Solve to find the number of sides

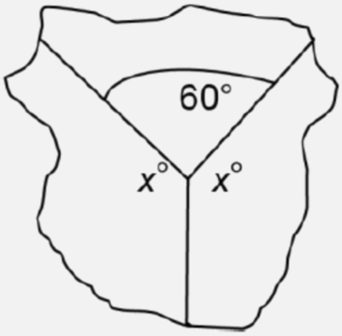
$$180(n-2) = 3960$$

$$180n - 360 = 3960$$

$$180n = 4320$$

$$n = 24$$

EXAMPLE:
Three regular polygons meet at a point, as shown in the diagram.
Find the number of sides of each of them.



Top polygon is an equilateral triangle (60, 60, 60) so 3 sides.

Find the size of each exterior angle

x = (360 – 60) ÷ 2 = 150

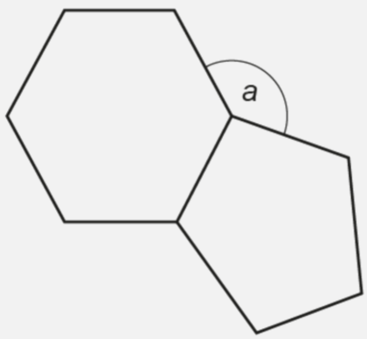
180 – 150 = 30

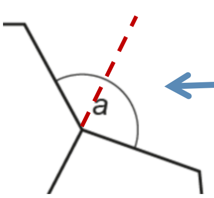
Angles around a point

360 ÷ 30 = 12

The polygons have 3, 12 and 12 sides.

EXAMPLE:
The diagram shows a regular pentagon joined to a regular hexagon.
Show that angle *a* is 132°.





One approach is to split angle a into two exterior angles

$$\text{exterior angle} = \frac{360}{n}$$

$$\text{pentagon, } \frac{360}{5} = 72$$

Find the size of each exterior angle

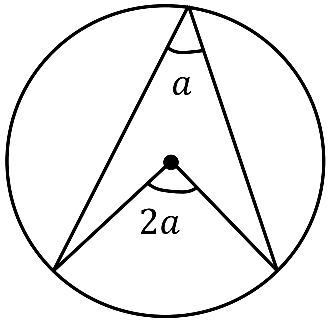
$$\text{hexagon, } \frac{360}{6} = 60$$

Add them together to get the required answer

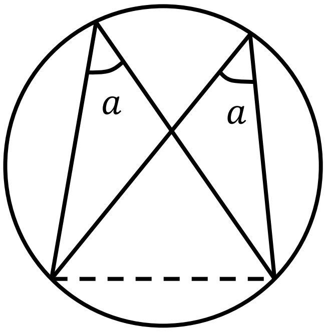
$$a = 72 + 60 = 132$$

circle theorems...

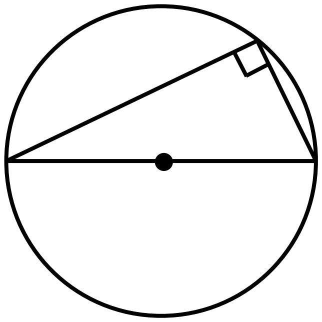
Circle theorems were mainly discovered and written down by Greek mathematicians, around the 5th century BC.



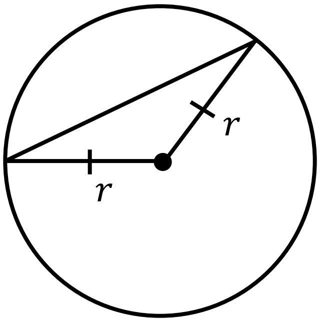
The angle at the centre is double the angle at the circumference



Angles in the same segment are equal

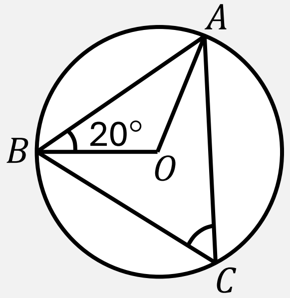


The angle in a semi-circle is 90°



(Watch out for isosceles triangles, formed from two radii: the ***base angles*** *are equal)*

EXAMPLE:
Calculate angle ACB.
Give a reason for each step of your working.

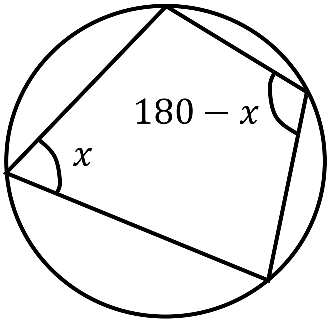


Angle OAB = 20°
because base angles in an isosceles triangle are equal.

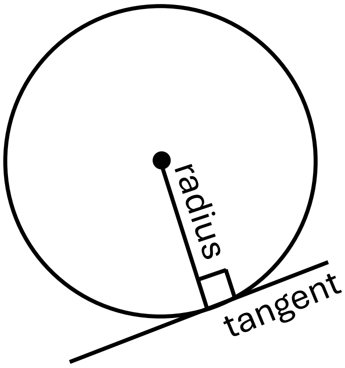
Angle AOB = 180 – (20 + 20) = 140°
because angles in a triangle add up to 180°.

Angle ACB = 140 x 2 = 280°
because the angle at the centre is double the angle at the circumference.

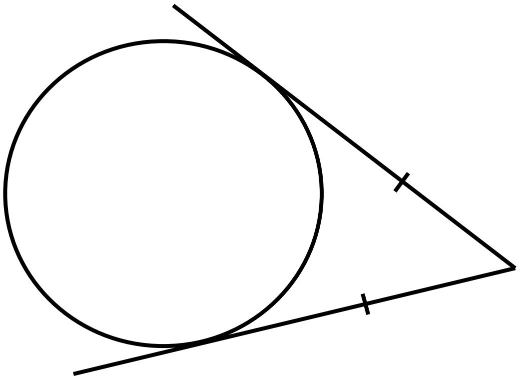
more theorems...



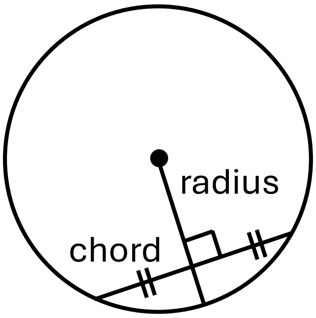
Opposite angles in a cyclic quadrilateral add up to 180°



A radius and a tangent meet at 90°

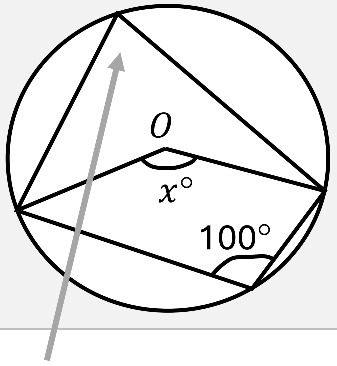


Tangents to a point are the same length



The perpendicular bisector of a chord is a radius

EXAMPLE: Find angle x.



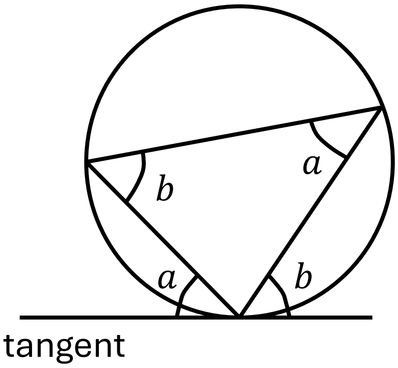
It can be helpful to mark angles on the diagram as you go along

Top angle = 180 – 100 = 80

x = 80 x 2 = 160°

You might not always be able to work out the angle that you want straight away. If in doubt, start by working out any angles that you can.

one more theorem...



The **Alternate Segment Theorem** states:
"The angle between a tangent to a circle and a chord is equal to the angle subtended by the chord in the alternate segment."

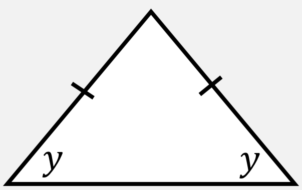
Put more simply, the angle a's in the diagram would be equal, and the angle b's would be too.

When we need to give a *reason*, we can just say "The Alternate Segment Theorem"

algebraic angles...

We can work with expressions for angles.

EXAMPLE: Find an expression for the third angle in this triangle, in terms of x and y.



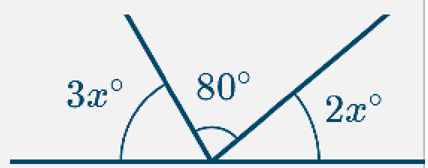
$$180 - (y + y)$$

$$= 180 - 2y$$

Angles in a triangle add up to 180°

angle equations...

EXAMPLE: By forming and solving an equation, find the value of x.



Angles on a straight line add up to 180°

$$3x + 80 + 2x = 180$$

Simplify the equation

$$5x + 80 = 180$$

$$5x = 100$$

then solve it

$$x = 20$$