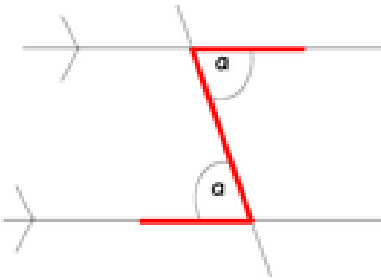




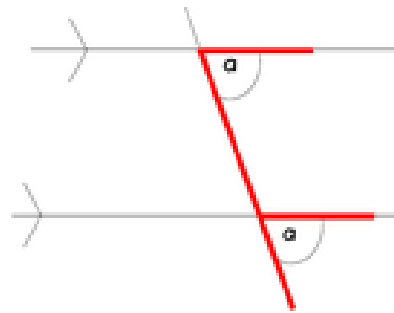
Unit 8: Angles

parallel lines...

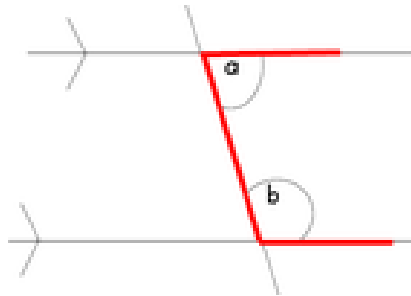
Alternate angles are equal.
(Inside the parallel lines – look for the ‘Z’ shape)



Corresponding angles are equal.
(Any matching pair of angles, e.g. both bottom right ones)

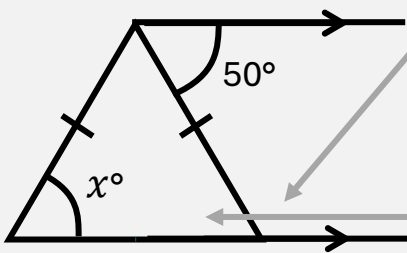


Co-interior angles add up to 180° .
(Look for the ‘C’ or ‘U’ shape)



EXAMPLE:

Find the size of angle x .
Give **reasons** to show how you decide.



$$180 - 50 = 130$$

Co-interior angles add up to 180°

$$180 - 130 = 50$$

Angles on a straight line add up to 180°

$$x = 50$$

Base angles in an isosceles triangle are equal.

“Give reasons” means write down the angle rules used.

exterior angles...

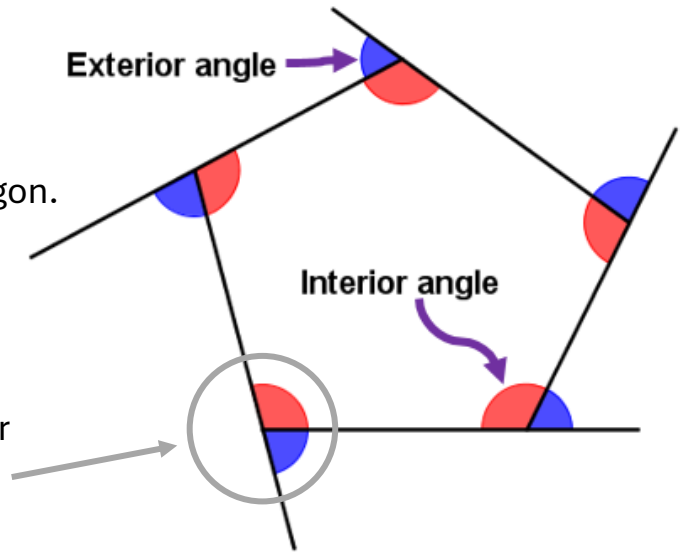
A **polygon** is a shape with straight sides.

A **regular** polygon has all equal sides, and all equal angles.

The diagram shows the **interior** and **exterior** angles of a polygon.

The exterior angles of any polygon always add up to 360°

Each pair of an interior and an exterior angle adds up to 180° , because they are angles on a straight line.



EXAMPLE: Calculate the size of the exterior angle in a regular 12-sided polygon.

$$360 \div 12 = 30^\circ$$

Exterior angles add up to 360° , divided by the number of sides/angles

EXAMPLE: The size of each interior angle in a regular polygon is 150° . Work out how many sides the polygon has.

$$180 - 150 = 30$$

Each exterior angle is 30° as a pair of an interior and exterior angle add up to 180°

$$360 \div 30 = 12 \text{ sides}$$

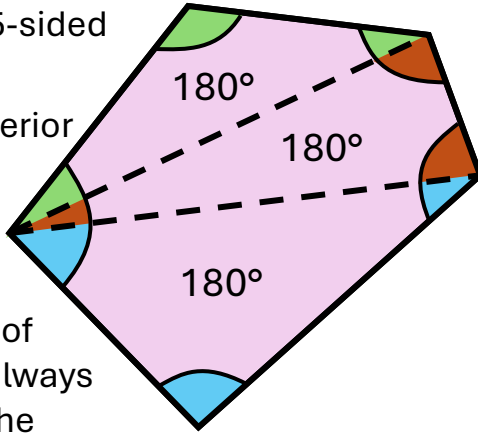
Exterior angles add up to 360, so there are 12 angles (and 12 sides)

interior angles...

The sum (total) of the interior angles can be found by dividing the polygon into triangles. Each triangle has 180° .

e.g. For this 5-sided polygon,

Sum of interior angles
 $= 3 \times 180$
 $= 540^\circ$



The number of triangles is always 2 less than the number of sides.

Formula:

The sum of the interior angles of an n -sided polygon is

$$(n - 2) \times 180$$

EXAMPLE: Find the sum of the interior angles of a decagon (10 sides)

$$\begin{aligned}(n - 2) \times 180 \\ (10 - 2) \times 180 \\ = 1440^\circ\end{aligned}$$

Substitute the number of sides into the formula

EXAMPLE: Work out the size of each interior angle in a regular octagon

$$\begin{aligned}(n - 2) \times 180 \\ (8 - 2) \times 180 \\ = 1080^\circ\end{aligned}$$

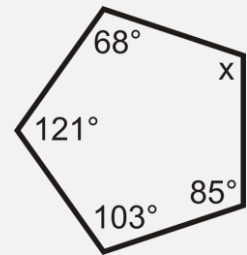
Find the sum of interior angles in an octagon

$$1080 \div 8 = 135^\circ$$

As it's regular, all 8 interior angles will be the same size

missing angles...

EXAMPLE: Calculate the size of angle x .



$$(5 - 2) \times 180 = 540$$

$$68 + 121 + 103 + 85 = 377$$

$$x = 540 - 377 = 163^\circ$$

Add up the other angles, then subtract from the expected total

advanced polygons...

EXAMPLE: An interior angle of a regular polygon is 11 times its exterior angle. Work out the number of sides of the polygon.

$$\text{exterior angle} = \frac{360}{n}$$

$$\text{interior angle} = \frac{(n-2) \times 180}{n}$$

$$\frac{(n-2) \times 180}{n} = 11 \times \frac{360}{n}$$

$$\frac{(n-2) \times 180}{n} = \frac{3960}{n}$$

$$180(n-2) = 3960$$

$$180n - 360 = 3960$$

$$180n = 4320$$

$$n = 24$$

Find expressions for the interior and exterior angles, with n as the unknown number of sides

Rewrite the statement in the question as an equation

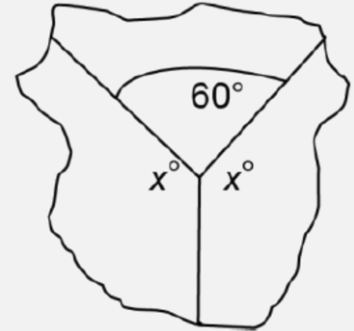
Solve to find the number of sides

Angles around a point

Find the size of each exterior angle

EXAMPLE:

Three regular polygons meet at a point, as shown in the diagram. Find the number of sides of each of them.



Top polygon is an equilateral triangle (60, 60, 60) so 3 sides.

$$x = (360 - 60) \div 2 = 150$$

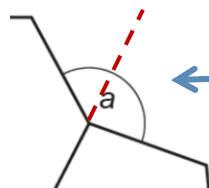
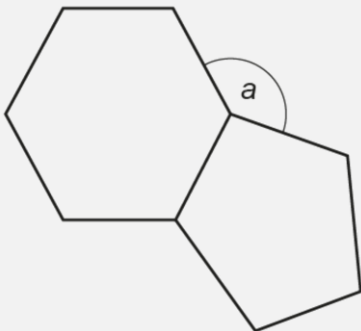
$$180 - 150 = 30$$

$$360 \div 30 = 12$$

The polygons have 3, 12 and 12 sides.

EXAMPLE:

The diagram shows a regular pentagon joined to a regular hexagon. Show that angle a is 132° .



$$\text{exterior angle} = \frac{360}{n}$$

$$\text{pentagon, } \frac{360}{5} = 72$$

$$\text{hexagon, } \frac{360}{6} = 60$$

$$a = 72 + 60 = 132$$

One approach is to split angle a into two exterior angles

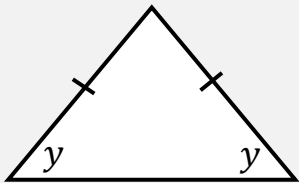
Find the size of each exterior angle

Add them together to get the required answer

algebraic angles...

We can work with expressions for angles.

EXAMPLE: Find an expression for the third angle in this triangle, in terms of x and y .

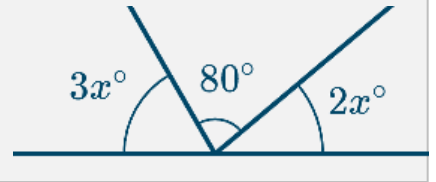


$$180 - (y + y) \\ = 180 - 2y$$

Angles in a triangle add up to 180°

angle equations...

EXAMPLE: By forming and solving an equation, find the value of x .



Angles on a straight line add up to 180°

$$3x + 80 + 2x = 180$$

$$5x + 80 = 180$$

$$5x = 100$$

$$x = 20$$

Simplify the equation

then solve it